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ABSTRACT

Because of the extensiveness of the course outline for Math 301 (Manitoba Department of Education), schools would have had to purchase several different textbooks to cover the material adequately. Therefore, a set of materials to supplement the guide was developed. The exercises, projects, and reviews contained in this package are keyed to the 301 portion of the "Mathematics 304, 301, 300 Pilot Guide (1982)." Arranged according to the modules of the guide, they are coded in the left-hand margin to the objectives of the Math 301 program. Review exercises follow either the sub-topics within each module or the entire unit. The first section consists of core topic exercises for algebra (graphing, algebraic equations, equation solving, variation), consumer mathematics (budgeting, taxation, insurance, mortgages), trigonometry, and statistics. The second section consists of optional exercises for: (1) annuities; (2) depreciation and distribution of overhead costs; (3) digital logic; (4) investments (stocks, bonds, real estate); (5) interpretation of graphs; (6) quadratic equations; and (7) permutations and combinations. In addition to the core and optional exercises, answer keys, notes to teachers (including instructional strategies), supplementary materials, and suggestions for student projects are provided. (Author/MP)

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province of manitoba
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EXERCISES TO ACCOMPANY MATHEMATICS 301

Manitoba Department of Education
1982

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USE OF THE DOCUMENT

DESCRIPTION

Because of the extensiveness of the course outline for Math 301, schools would have to purchase several different textbooks in order to cover the material adequately. As this would have been unacceptable to both schools and teachers, the Curriculum Development and Implementation Branch commissioned a set of materials to supplement the guide and to assist the Mathematics 301 pilot teachers during the 1982-83 school year.

During the summer of 1982, a number of 01 Mathematics Development-Writing Committee members and Math 301 pilot teachers met to prepare this series of sample exercises, questions and projects. The object has been to help teachers confronted with the task of implementing a new program without a text.

FORMAT

The exercises, projects and reviews contained in this package are keyed to the Mathematics 301 portion of the Mathematics 304, 301, 300 Pilot Guide (1982). Arranged according to the modules of the guide, they are coded in the left-hand margin to the objectives of the Math 301 program. Review exercises follow either the sub-topics within each module or the entire unit. Answers for all exercises are provided at the end of each unit to assist both students and teachers. Where necessary, blank forms pertaining to the examples under consideration have been provided for teacher use.

C.1. ALGEBRA

A Graphing

A.1

1. Zero an ordinary pan-balance with a container on one side. Add marbles (uniform in size) to the container, recording the mass after each marble.

2. Repeat the above experiment using different substances (example, scoops of sand, beads, iron shot, etc.).

3. Stack 10 to 12 texts (uniform in size) on top of each other. Record the height of the stack after each book is added.

A.2

1. Graph the data obtained in the previous experiments.

2. Graph the following data:

a)

Number of Apples	0	1	2	3	4	5	6	7	8	9	10
Price in Cents	0	35	70	105	140	175	210	245	280	315	350

Cost of Apples in a Cafeteria

b)

Distance in km	0	10	20	30	40	50	60	70	80	90	100	120
Time in minutes	0	10	20	30	40	50	60	70	80	90	100	120

A car travelling at a uniform speed of 60 km/hr.

c)

Number of bottles	0	2	4	6	8	10	12	14	16	18	20
Capacity in mL	0	1500	3000	4500	6000	7500	9000	10 500	12 000	13 500	15 000

The capacity of one bottle of soda is 750 mL

3. In each of the following plot the graph to illustrate the given information.

a) The cost per pencil is 50 cents (from o to 12 pencils).
b) The initial temperature is $0^{o}C$ and the temperature increases $8^{o}C$ each half hour over an interval of 5 hours.
c) The distance travelled from 0 to the 10th second by an object travelling at a uniform rate of 20 m per second.

A-4

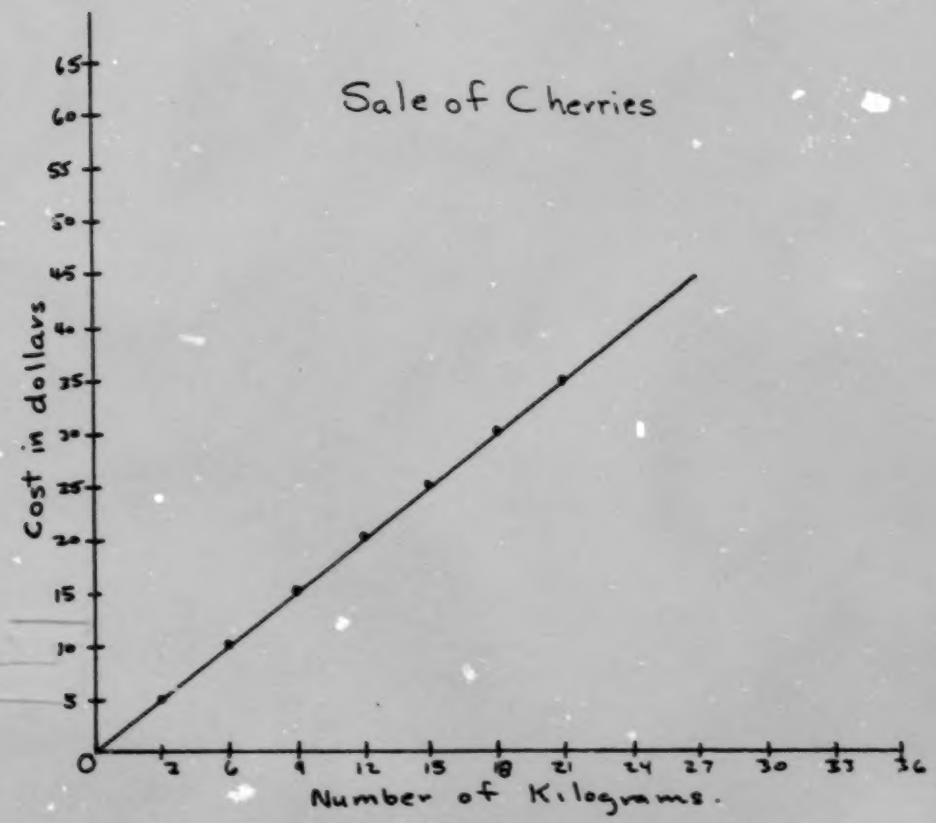
1. Using the information from question 2(b) of A-2, how many km would the car have travelled after (a) 15 minutes?
   (b) 93 minutes?
   (c) 150 minutes?

2. Using the information from question 2(c) of A-2, what is the capacity of (a) 7 bottles of soda?
   (b) 15 bottles of soda?
   (c) 24 bottles of soda?
   (d) 21 bottles of soda?

3. Using the information from question 3(b) of A-2, what would the temperature be after (a) 45 minutes?
   (b) 2 hours, 15 minutes?
   (c) 5 1/2 hours?

4. Using the information from question 3(c) of A-2, how far will the object have travelled after (a) 7.5 seconds?
   (b) 12 seconds?

5.



a) How much will 6 kg of cherries cost?
b) How much will 10.5 kg of cherries cost?
c) How much will 7 kg of cherries cost (approx.)?
d) How much will 20 kg of cherries cost (approx.)?
e) How much will 36 kg cost?

A-5

1. Write an equation to express the data from questions #1 and #3 of section A-1.

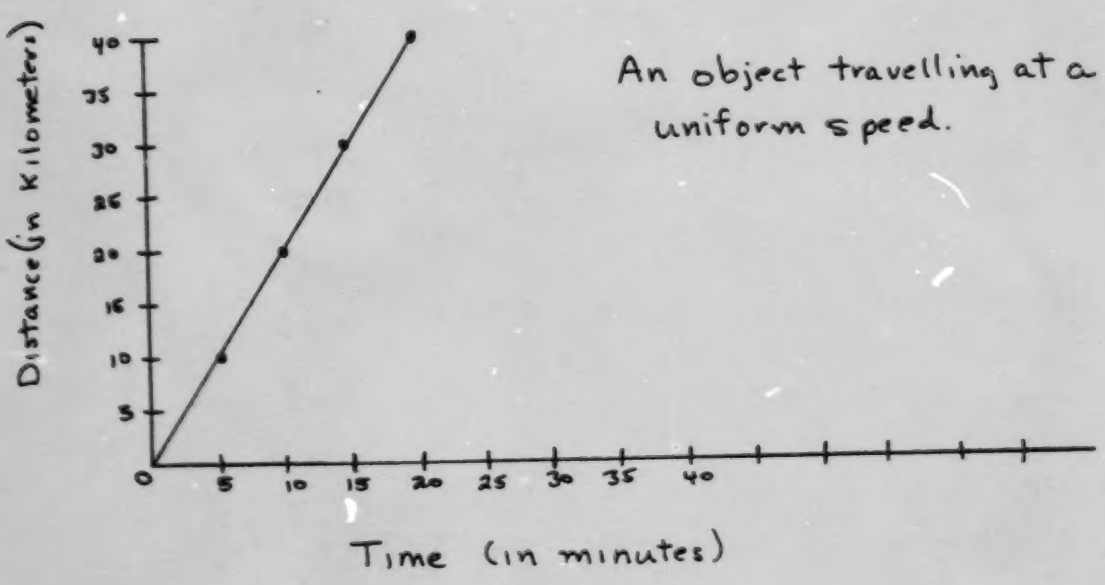
2. Write an equation to express the data from question #20(a) to (c) of section A-2.

3. Write an equation to express the data from question #3(a) to (c) of section A-2.

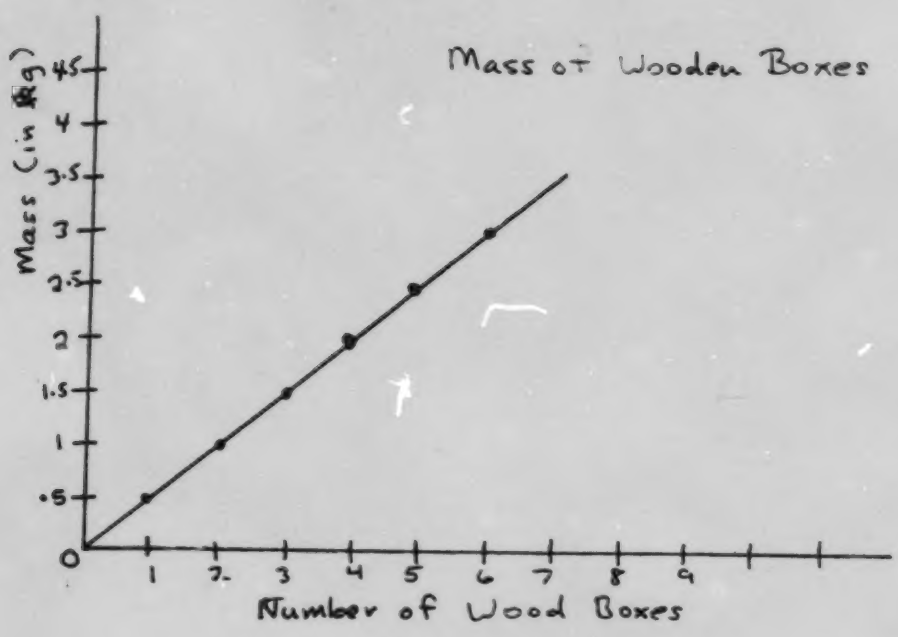
4. Write an equation to express the following data:

Cost (in dollars)	0	16	32	48	64	80	96	112
No. of Footballs	0	1	2	3	4	5	6	7

5. Write an equation to express the data represented by the following graph:



6. Write an equation to express the data represented by the following graph:



A-6 1. Graph the following linear relations.

a) $y = 3x$
b) $5x = y$
c) $y = x$
d) $y = 0.4(x)$
e) $y = 1.25(x)$
f) $y = 1/2\ x$
g) $y = \frac{3x}{4}$
h) $y = \frac{x}{3}$
i) $2y = 12x$
j) $5y = x$
k) $y - 2x = 0$
l) $4x - y = 0$
m) $4y - 8x = 0$
n) $5x - 3y = 0$
o) $3y - 2x = 0$

B Partial Linear Relations (4 periods)

B-1 1. Obtaining data through experimentation.

a) Given a graduated cylinder containing a stone, have students add equal amounts of water (i.e., 100 ml at a time). Have students measure the height of the water columns.

b) Given a student standing on a kilogram scale, have the student hold an increasing number of books, and measure mass of student and books, as each book is added.

c) Given a beaker (1 L) of water at room temperature (21°C), allow a bunsen burner to heat water and record temperature at frequent, regular intervals.

d) Calculate the cost of a volleyball trip for up to 10 persons given a $400 rental fee for a van and $20 expenses for each person.

B-2 1. Graph the following sets of data.

a) Weight of student alone 68 kg.
" " " with 1 book 69.5 kg
" " " with 2 books 71 kg
" " " with 3 books 72.5 kg
" " " with 4 books 74 kg
" " " with 5 books 75.5 kg
" " " with 6 books 77 kg

b) Water in beaker at room temperature 21°C
Water after 2 minutes 23°C
Water after 4 minutes 25°C
Water after 6 minutes 27°C
Water after 8 minutes 29°C
Water after 10 minutes 31°C
Water after 12 minutes 33°C
Water after 14 minutes 35°C

c) Graph data from 1(d)

B-4 1. a) Using data collected in B-2(a):
(i) What is the weight of each book?
(ii) What would be the weight of the same person with 12 books?

b) Using data collected in B-2(b):
(i) What would the water temperature be after 5 minutes?
(ii) What would the water temperature be after 9 1/2 minutes?
(iii) What would the water temperature be after 20 minutes?
(iv) What is the heat gain per minute?

c) Using data from B-1(d):
(i) What would the total cost of the trip be with 12 people?
(ii) What would the cost per person be for a trip with 8 people?
(iii) What would the cost per person be for a trip with 6 people?

B-5 1. a) Determine an equation to express the data obtained in B-2(b), B-2(c) and B-1(d).

b) The cost of a carpentry job includes $45 for power hammer rental plus 100 nails. Additional nails are purchased according to the following schedule:

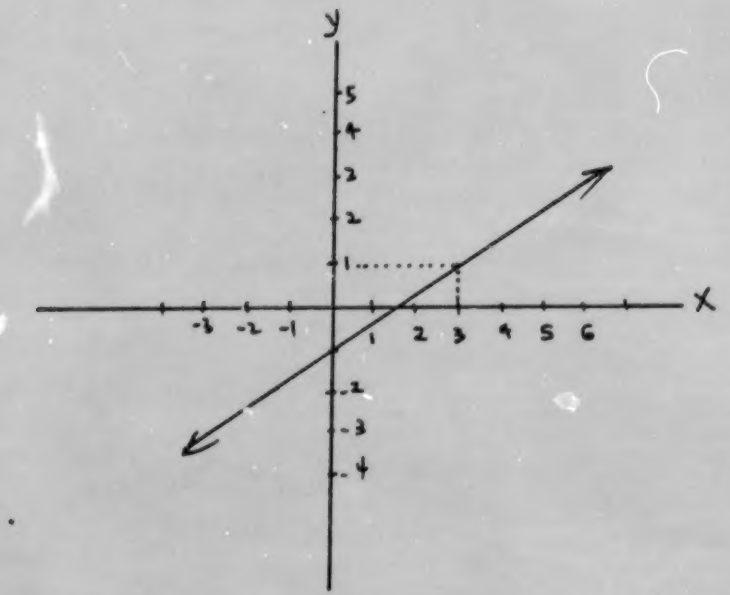
101-200	$2.95
201-300	5.90
301-400	8.85
401-500	11.80
501-600	14.75
601-700	17.70
701-800	20.65
801-900	23.60
901-1000	26.55

Write an equation relating total cost to the number of nails required. Estimate the cost of renting the hammer and using 750 nails.

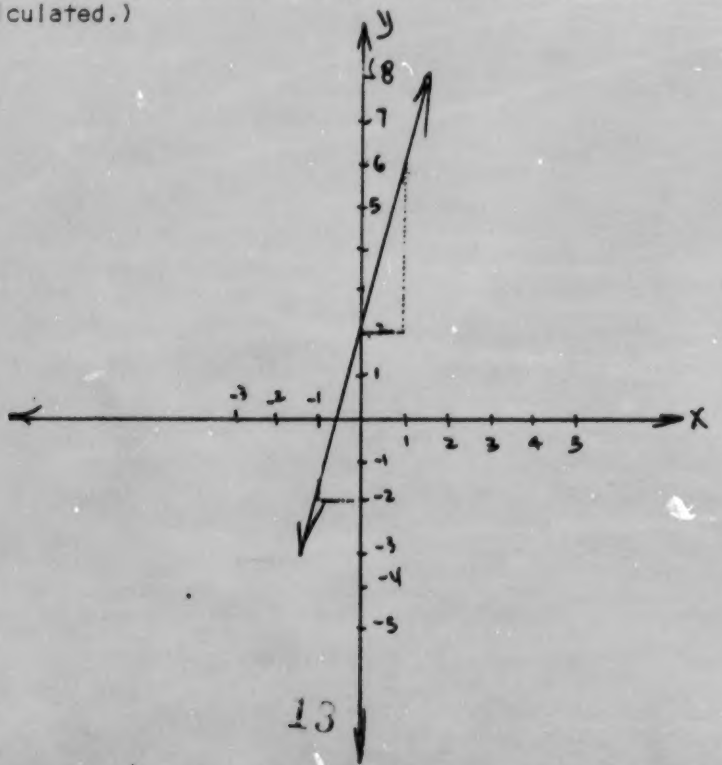
B-6 1. a) Given the following equations:
(i) rewrite each in the form $y = mx + b$
(ii) graph each equation

1) $4y = 3x - 10$
2) $2y + 4x = 8$
3) $6x - y = 3$
4) $8x + 2y + 2 = 0$

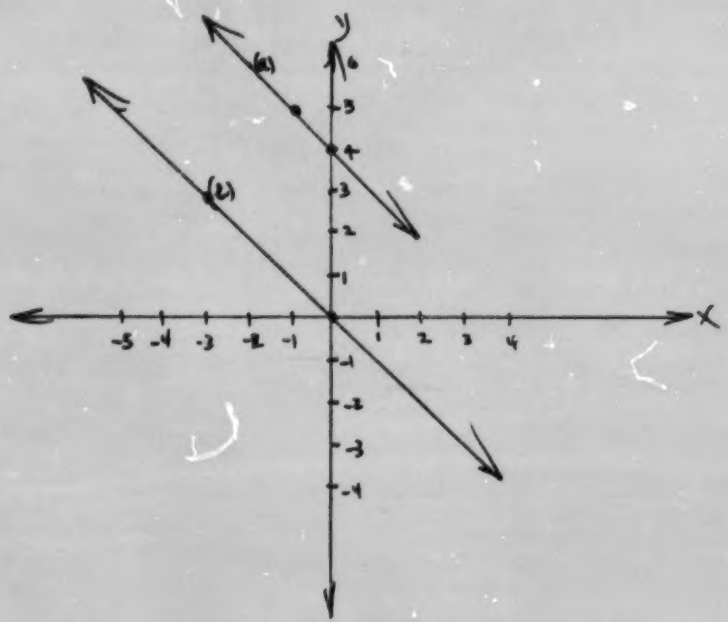
b) Write an equation in the form y = mx + b for this graph.



Write an equation in the form y = mx + b for this graph. (NOTE: There are 2 places on this graph where slope could easily be calculated.)



i) Write equations for both lines shown.
ii) Sketch the graph for a line with the same slope as those shown and with a y- intercept of -4.
iii) These lines are all _____.



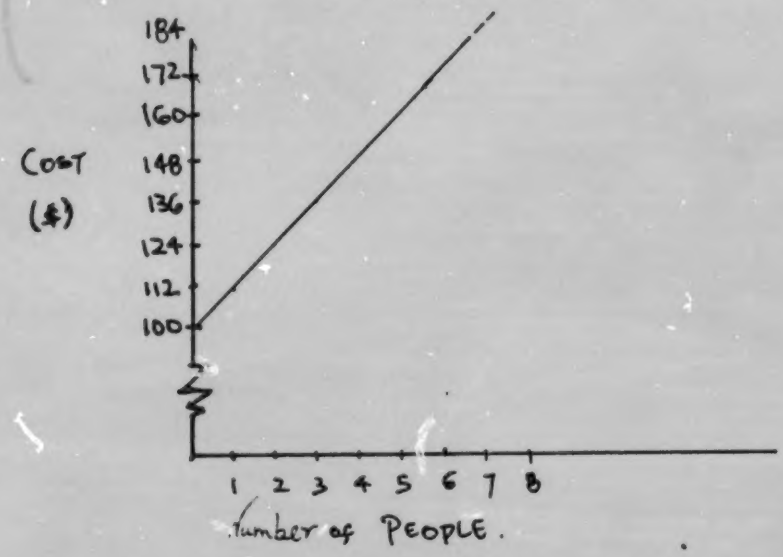
B-7 1. a) Write an equation in the form y = mx + b for the straight line whose slope is -2 and whose y- intercept is -5.

b) Write an equation in the form y = mx + b for the straight line whose slope is 4/3 and which passes through the point (0, -3).

c) Write an equation in the form y = mx + b for the line passing through points (0,4) and (-4,-5).

d) Write an equation in the form y = mx + b for the line passing through points (6,2) and (8,-3).

2. a) The accompanying chart indicates how the cost of providing a dinner is related to the number of people. Determine the equation for the straight line relationship.



b) At a Winnipeg car rental agency, a rate of $28 per weekend plus 8¢/km is offered. Graph the straight line and write an equation in the form y = mx + b for the line. From your equation, what would be the cost of the car for a weekend if the car was driven for 650 km? 1000 km?

c) In a certain form of test, a student has an initial mark of 100 with marks deducted for every error. If marks are deducted at a rate of 3 for every mistake, produce a graph of the straight line that represents this situation. Write an equation that properly expresses this relationship. From your equation, calculate the minimum number of errors which will produce a failure (less than 50).

C Non-Linear Relations (2 periods)

1. a) The space shuttle, upon lift-off, is very slow. However, as it burns tonnes of fuel it accelerates to thousands of kilometres per hour due principally to the fact that it is free of gravity's pull. The following data represents velocity against time.

Time (seconds)	1	2	3	4	5	6	7	8	9	10	11	12
Velocity (m/sec)	.21	.48	1.1	1.6	2.2	3.4	5	6.3	8.3	10.5	14.3	20

Plot the points and join them with a smooth curve.

b) The formula $d = 1/2\ at^2$, describes the relationship of an object in free fall (wind resistance ignored). The distance fallen (d) equals the product of one-half the constant of acceleration (9.8) and the time required. For example, the distance dropped after 2 seconds is:

$$d = 1/2(9.8)(2)^2$$
$$= 1/2(9.8)(4)$$
$$= 19.6$$

Given an object in free fall for 6 seconds, prepare a graph of the distance fallen vs. time.

c) For any object thrown horizontally from a tower, the lateral velocity remains constant until the object hits the ground (ignoring air resistance). However, due to gravity's pull, the object begins to accelerate downwards upon release. Present a graph to show the behaviour of a ball thrown horizontally at 10 m/sec which remains in the air for 4 seconds. NOTE: downward movement is expressed by $d = 1/2\ at^2$ where $a = 9.8$.

2. a) From the data in 1(a), how long did it take the shuttle to reach a velocity of 15 m/sec? 30 m/sec?

b) Based on the data in 1(b), how far do you expect something to drop after 7 seconds? 10 seconds?

c) From the data in 1(c), how far horizontally would the object travel if it was air borne for 8 seconds?

B. Algebraic Operations

B.1

1. Write the value of each expression:

a) 4^2
b) 5^3
c) 2^6
d) 1^7
e) 3^1
f) $(-2)^4$
g) $(-3)^3$
h) $(-4)^2$

2. Simplify and write each answer in exponential form:

a) $3^3 \cdot 3^2$
b) $4^5 \cdot 4^6$
c) $2^7 \cdot 2^6$
d) $7^5 \cdot 7^8$
e) $5^6 \cdot 5^3 \cdot 5^2$
f) $2^7 \cdot 2^2 \cdot 2^3$
g) $3^4 \cdot 3^5 \cdot 3^3$
h) $8^2 \cdot 8^3 \cdot 8^3$
i) $x^3 \cdot x^5$
j) $m^4 \cdot m^7$
k) $t^3 \cdot t^2 \cdot t^4$
l) $b^6 \cdot b^3 \cdot b^9$

3. Simplify (evaluate each answer, where possible):

a) $\dfrac{4^4}{6^2}$
b) $\dfrac{10^{22}}{10^{20}}$
c) $\dfrac{11^3}{11^3}$
d) $\dfrac{7^7}{7^4}$
e) $\dfrac{2^{15}}{2^{10}}$
f) $\dfrac{3^3 \cdot 3^4}{3^5}$
g) $\dfrac{5^6 \cdot 5^3}{5^8}$
h) $\dfrac{3^2 \cdot 3^3 \cdot 3^6}{3^4 \cdot 3^5}$
i) $\dfrac{x^9}{x^5}$
j) $\dfrac{a^{16}}{a^9}$
k) $\dfrac{n^3 \cdot n^5}{n^4}$
l) $\dfrac{w^9 \cdot w^8}{w^4 \cdot w^3}$

4. Write the value of each:

a) 2^{-3}
b) 3^{-2}
c) $4^{-3} \cdot 4^5$
d) $2^6 \cdot 2^{-8}$
e) $4^6 \cdot 4^{-4} \cdot 4^1$
f) $2^8 \cdot 2^{-5} \cdot 2^{-2}$
g) $\dfrac{3^6}{3^{-2}}$
h) $\dfrac{2^{-4}}{2^{-5}}$

5. Write the value of each:

a) 2^{-3}
b) 3^{-2}
c) 4^{-3}
d) 8^{-2}
e) $4^6 \cdot 4^{-4} \cdot 4^1$
f) $2^8 \cdot 2^{-5} \cdot 2^{-2}$
g) $\dfrac{3^6}{3^{-2}}$
h) $\dfrac{2^{-4}}{2^{-5}}$
i) $\dfrac{3^{-7}}{3^{-6}}$
j) $\dfrac{4^5}{4^7}$
k) $\dfrac{2^6 \cdot 2^{-8}}{2^3}$
l) $\dfrac{10^2 \cdot 10^3}{10^5 \cdot 10^4}$
m) $\dfrac{9^3 \cdot 9^{-5}}{9^{-6}}$
n) $\dfrac{4^{-2} \cdot 4^3 \cdot 4^{-5}}{4^6 \cdot 4^{-4}}$
o) $\dfrac{(2 \cdot 3)^{-5}}{(3 \cdot 2)^{-6}}$
p) $\dfrac{3^{-5} \cdot 3^{-6}}{3^{-7} \cdot 3^{-3}}$

B.2

1. Simplify each:

a) $6a + 7a$
b) $8x + 5x + 3x$
c) $9m - 5m + 3m$
d) $6t - 8t - 5t$
e) $(5x + 6) + (2x - 3)$
f) $(23 - 17x) + (13 - 9x)$
g) $(6a^2 + 3a) - (7 + 5a^2) - (8a - 6)$
h) $(7x - 9) - (5x + 3)$
i) $(19 + 7x) - (18 - 8x)$
j) $(5x + 8) - (-3x - 7)$
k) $(8x + 3) + (7x - 9) + (-6x - 7)$
l) $(4x - 6) - (-3x + 7) - (5x - 8)$
m) $(3x + 2) - (3 - 4x) + 2x$
n) $(3a^2 - a) + (4a - 5) + (2a - 2a^2)$

2. Simplify each:

a) $2(7x)$
b) $3(4x^3)$
c) $4(-5a^4)$
d) $(-6)(-8b^5)$
e) $(-3)(7m^4)$
f) $(2x)(3x^4)$
g) $(-5a^3)(6a^5)$
h) $(-7m^4)(-8m^8)$
i) $4(3x + 5)$
j) $6(5x - 7)$
k) $-7(3m^2 + 4m)$
l) $6t(4t^3 + 2t^2)$
m) $(x + 3)(x + 5)$
n) $(a - 5)(a + 6)$
o) $(t + 7)(t + 7)$
p) $(3b + 2)(2b - 4)$
q) $(2m - 5)(5m + 4)$
r) $(3x - 5)(3x + 5)$

3. Simplify each:

a) $5(a - 2) + 3(2a + 4)$
b) $3(2b + 6) - 4(3b - 8)$
c) $5(3m + 7) + 6(-2m - 5)$
d) $4(-5m - 9) - 6(4m - 8)$
e) $3t(2t + 1) + 2t(3 - 4t)$
f) $\dfrac{35x^5}{7x^2}$
g) $\dfrac{50m^{11}}{25m^8}$
h) $\dfrac{8z^{15}}{4z^9}$
i) $\dfrac{32m^4 + 8m^2}{4m}$
j) $\dfrac{26w^5 + 3qw^4}{13w^2}$
k) $\dfrac{27m^8 - 42m^5 + 6qm^2}{3m^2}$

4. Solve each problem:

a) What must be added to $5x + 3$ to produce zero?

b) What is the perimeter of a square whose side is $3x^2 = 5x + 2$?

c) What is the perimeter of an equilateral triangle if a side is $7a - 2$?

d) Find the perimeter of a triangle whose sides are $(3m - 2)$, $(4m + 6)$ and $8m$.

e) What must be added to $a^2 - 8$ to produce $a^2 + 8$?

f) Subtract $5x - 3$ from the sum of $8x + 5$ and $7 - 3x$.

g) From $(10x^2 - 3)$ subtract the sum of $(3x^2 + 4)$ and $(x^2 + 3)$.

h) The perimeter of a triangle is $8x + 5$. If two of the sides are $3x - 6$ and $4x + 8$, find the third side.

i) Find the area of a rectangle if the length is $a + 5$ and the width is $a - 7$.

j) The length of a rectangle is designated by $3x + 6$, the width is designated by $5x - 2$. Write an expression for its perimeter.

k) A triangle has its base designated by $6x + 2$ and its altitude designated by $3x - 5$. Write an expression for its area.

B.3 1. Write each number in scientific notation:

a) 36 000

b) 7300

c) 82 000 000

d) 1 250 000 000

e) 83 million

f) 23 billion

g) one hundred billion billion

h) the product of (one million)(one million)

i) 30^4

j) $20 \times 10^3 \times 10^4 \times 10^6$

B.4 1. Write each number in scientific notation:

a) 0.0034
b) 0.000 56
c) 0.000 000 08
d) 0.0094
e) 0.000 000 705
f) 0.958
g) 0.000 006 753
h) thirteen ten-thousandths
i) 40^{-2}
j) $(2.3 \times 10^{-6}) \cdot (3.2 \times 10^{-8})$

B.5 1. Write the following in expanded notation:

a) 3.46×10^{7}
b) 1.15×10^{-3}
c) 8.6×10^{4}
d) 30.4×10^{-6}
e) 8.88×10^{-4}
f) 1.009×10^{8}
g) 4.5×10^{5}
h) 3.3×10^{-8}
i) 1.00×10^{6}
j) product of $(3.4 \times 10^{-8})(2.3 \times 10^{14})$

C. Equations

C.1 1. Solve each equation:

a) $a + 5 = 10$
b) $b + 6 = 15$
c) $c - 9 = 23$
d) $8 - d = 15$
e) $6 + e = -9$
f) $4f = 20$
g) $2q = 21$
h) $3h = 52$
i) $6a + 4 = 28$
j) $7b + 8 = 36$
k) $3c - 9 = 18$
l) $2d - 5 = 9 + 6$
m) $3m = -16$
n) $8n = 7n + 1$
o) $5a + 3 = 2a - 7$
p) $6a + 1 = 5a$
q) $4m - 2 = 3m - 5$
r) $6x + 5 = 9x + 5$
s) $4y + 528 = 0$
t) $a + 3a = 7a + 15$
u) $6b + 1 + 2b = 4 + 9b$
v) $9 - 2c = 30 - c$
w) $72 - 3n = 84 + 4n$
x) $13 + 5a - 6a = 23 + 7a + 4$
y) $6x - 126 = 0$
z) $2a - 3 + 5a = 4a + 3a$

2. Solve each equation:

a) $3(x + 5) + 4(x + 5) = 21$
b) $2(a + 4) + 3(a + 4) = 40$
c) $15 + (b + 3) + 2(b + 4) = 29$
d) $23 + 2(c-7) - 3(c + 4) = 32$
e) $4(d + 3) - 2(d + 7) = 3(d-6)$
f) $4(e + 11) - 5(e - 8) = 0$
g) $7(f - 16) = 3(f + 4)$
h) $2(5a - 4) - 3(6a + 5) - 10a = 5(3a - 16)$
i) $(z + 5)(x - 4) = (x + 3)(x - 7)$

3. Solve each equation:

a) $\frac{1}{3}a = 27$

b) $\frac{2}{5}b = 4$

c) $\frac{1}{6}c = \frac{1}{2}$

d) $\frac{3}{4}d = -2$

e) $\frac{3}{7}a = -6$

f) $\frac{2}{9}b = \frac{1}{9}$

g) $\frac{3}{7} - \frac{1}{2}c = \frac{2}{7}$

h) $\frac{4}{9} + \frac{2}{3}d = 1$

i) $\frac{a}{2} - \frac{3a}{5} = 2$

j) $\frac{1}{4}b - 3 = -2$

k) $\frac{2}{3}c + 4 = 6$

l) $\frac{d}{4} - 2 = \frac{d}{6} + \frac{d}{8}$

m) $\frac{3a}{5} - \frac{a}{2} = \frac{2}{5} + \frac{3a}{2}$

n) $\frac{3}{2} - \frac{b}{3} = 5 + \frac{b}{6}$

o) $-4 - \frac{1}{4}c = \frac{1}{16}c + 1$

p) $\frac{3d}{2} + \frac{5}{24} = \frac{5d}{8} + \frac{13}{12}$

q) $\frac{a}{6} - \frac{a}{9} + \frac{1}{3} = \frac{3a}{2} - \frac{4}{9}$

4. Solve each equation:

a) $\frac{a + 2}{3} = \frac{2a - 7}{5}$

b) $\frac{4b - 9}{2} = \frac{5 - 3b}{9} - \frac{7}{18}$

c) $\frac{3 - c}{7} = \frac{-6 - 5c}{2}$

d) $\frac{d - 7}{2} - \frac{1}{3} = 1 + \frac{d + 9}{9}$

e) $\frac{3}{4}(3a + 1) - \frac{8}{7}(4a - 8) = -11$

f) $\frac{3b - 10}{7} = \frac{4 - 2b}{3} + 1$

g) $4(2c - 3) + \frac{1}{2}(6 - 7c) = 8 - \frac{2}{3}(4c + 5)$

h) $\frac{2}{3}(d + 5) - \frac{3}{4}(2d - 3) = \frac{1}{2}(6 - 2d) + \frac{1}{3}(7 + 3d)$

i) $\frac{7}{5}(a - 3) + \frac{4}{3}(2a + 1) = 0$

C.2 1. For each problem, set up an equation in one variable and then solve.

a) Find two consecutive integers whose sum is 35.
b) Two more than three times a number is 23.
c) Five less the four times a number is 23.
d) Find three consecutive integers whose sum is 63.
e) John is 18 years older than Derek. The sum of their ages is 46.
f) The perimeter of a rectangle is 400. Find the length if it is 2 more than twice the width.
g) Georgina is 10 years older than Marianne. In 3 years she will be twice as old. How old is Georgina now?
h) Pamela is 5 years older than Albertine. Francine is twice as old as Albertine. The sum of their ages is 65. How old is each?
i) Three times one number equals another number. If the sum of the 2 numbers is 64, what is each number?
j) Find each of three consecutive odd numbers if their sum is 237.
k) Find each of three consecutive even numbers if their sum is 378.
l) Find a number such that 2/3 of the number decreased by 9 is 35.
m) Two cars start from the same town and travel in opposite directions. One travels 90 km/hr. and the other travels 100 km/hr. How many hours will have passed when they are 980 km apart?
n) Joanne has $3.78 in coins, consisting of dimes, nickels and pennies. There are twice as many pennies as nickels, and the number of dimes equals the number of pennies. How many of each are there?
o) The body of a certain missile is 12 times the length of the nose cone. If the total length of the missile is 38 metres, find the length of the nose cone.
p) At Winnipeg International Airport, Airline A has 1 1/2 times as many planes as Airline B. Airline C has 1/3 as many planes as A and B together. All three together have 180 planes. How many planes does each have?
q) The sum of 2 numbers is 56. Twice the larger decreased by three times the smaller is 16 more than the smaller number. Find each number.

C.3-A 1. Find 3 ordered pairs which satisfy each of the following linear equations:

a) $x + y = 13$
b) $2x + y = 9$
c) $3x - 2y = 4$
d) $5x - y = 12$
e) $x + y = -15$
f) $4x + 5y = -8$
g) $3y - 4x = 9$
h) $y + 6x = 15$

2. For each linear equation write the ordered pair solutions which have x-values of -2, -1, 0, 1 and 2.

a) $2x + 3y = 6$
b) $3x - y = 8$
c) $2y - 5x = -15$
d) $-4y - 4x = 4$

C.3-B 1. Decide whether each ordered pair is a solution for the given system of equations:

a) $2x - 3y = -1$, $x + 4y = 16$ — (4,3), (3,4)

b) $x - 3y = 8$, $2x + y = 11$ — (5,-1), (-1,5)

c) $y - 2x = 6$, $x + 3y = -7$ — (-6,3), (3,-6)

d) $2x - 3y - 1$, $3x + 2y = -18$ — (-4,-3), (4,3)

e) $2x + y = 8$, $4x - 9y = 5$ — (7/2,1), (2,4)

f) $3x + 2y = 1$, $2x - 3y = -8$ — (-2,1), (-1,2)

C.3-C 1. Graph the following systems of equations and determine the solution (point of intersection) for each system.

a) $x - y = 5$, $3x + 2y = 25$

b) $x + 4y = -1$, $2x - 3y = 9$

c) $3x - 2y = 0$, $4x + y = -11$

d) $5x + 6y = -3$, $2x - 2y = -10$

e) $3x + 5y = -21$
$x - 4y = -7$

f) $4x - 6y = 3$
$3x + 8y = -4$

g) $5x - 3y = 8$
$-10x + 6y = 4$

h) $x + 3y = 4$
$2x + 6y = 8$

i) $-x + 2y = 2$
$x - 2y = 6$

j) $2x - 3y = 4$
$-2x + 3y = -4$

k) $\frac{2x + 3}{6} - y = \frac{1}{6}$

$3x + 2y = 16$

l) $x = 4$
$y = 5$

m) $y = -3$
$x + 2 = y$

n) $x = 7$
$2x - y = 10$

C.3-D 1. Solve each of the following systems of equations, using an algebraic method:

a) $3x - 4y = -6$
$3x + 2y = 12$

b) $4x + 3y = -5$
$x + 3y = -8$

c) $x + 2y = -6$
$2x + y = -6$

d) $5x + 7y = 5$
$x - 3y = 1$

e) $3x - 2y = 10$
$x + y = -5$

f) $8x - 5y = 0$
$5x - 8y = 0$

g) $6x + 9y = 6$
$8x - 3y = 3$

h) $4x - 7y = 0$

$2x + \frac{7}{2}y = 1$

i) $6x + 8y = 6$
$3x - 12y = -1$

j) $7x - 9y = -6$
$-2x + 3y = +6$

k) $4x + y + 8 = 0$
$5x - 2y + 23 = 0$

l) $\frac{1}{4}x - 2y = 1$
$3x + 4y = 12$

m) $x - \frac{y - 2}{3} = 0$

$\frac{2}{10}x + \frac{3}{10}y = 5$

n) $.3x - .4y = 5$
$2x + 5y = 52$

o) $3x - 4y - 1 = 3(2x - y + 2)$
$2(4x + 2y + 3) = 7x + y + 9$

p) $2(2x + 5y - 7) = 13y - x - 9$
$3(5x - 2y - 1) = 3(3x + 2y) - 13(y - 2)$

q) $y = -6$
$3x + 4y = -12$

r) $x = 7$
$2x - 5y = -1$

s) $\frac{x + y}{5} + \frac{y - x}{5} = 4$

$\frac{2x + y}{10} - \frac{x + 2y}{5} = -3$

t) $\frac{2x + y}{7} + \frac{x - y}{10} = 39$

$\frac{x - 2y}{5} - \frac{3x - 2y}{7} = -32$

u) $\frac{x}{8} - \frac{4}{9} = -1$

$\frac{2x}{3} + \frac{3y}{4} = 43$

v) $x = 15$
$y = -7$

C.4 1. Solve each problem by first writing a system of equations to represent the problem.

a) The sum of two numbers is 28. Their difference is 8. Find the numbers.
b) The sum of two numbers is 250. Their difference is 68. Find the numbers.
c) The sum of two numbers is 38. Twice the smaller added to three times the larger is 106. Find the numbers.
d) The difference of 2 positive numbers is 24. Three times one number added to twice the other is 92.
e) The manager and his assistant are to share a bonus of $1200. The manager's share of the bonus is three times the amount of the assistant's share. How much does each receive?
f) 340 people bought tickets to the high school drama. Each adult paid $3.00 and each student paid $1.25. The total collected was $708.50. How many adults attended the drama?
g) A child's piggy bank contained $5.24 in nickels and pennies. The number of pennies exceeded the number of nickels by 176. How many of each were there?
h) The accountant of A. B. Company made a deposit in the local bank consisting only of one-dollar bills and five-dollar bills. There were 357 bills totalling $1125. How many of each were there?
i) Two trains leave Winnipeg, one going west, the other going east. The second train is travelling 15 km/hr. faster than the first. After 5 hours the trains are 775 km apart. What is the speed of each?
j) The cost of 3 dozen eggs and 4 pounds of butter is $12.35, while the cost of 4 dozen eggs and 3 pounds of butter is $11.45. What is the cost of 2 dozen eggs and 2 pounds of butter?
k) The perimeter of a rectangle is 32 centimetres. The length is 2 centimetres less than twice the width. Find the length of the diagonal of the rectangle.
l) A merchant wishes to prepare a mixture of 20 kg of candy to sell at $4.50 per kilogram. To do this he mixes some candy which sells for $3.75 per kilogram with candy that sells for $6.00 per kilogram. How much of each should he use?
m) A man bought $16.80 worth of stamps. Three times the number of 60 cent stamps was equivalent to twice the number of 30 cent stamps. How many of each did he buy?
n) A number, less than 100, has its tens-digit 2 more than its units-digit. If the number with its digits reversed is subtracted from the number itself, the difference is 2 more than twice the sum of the digits. What is the number?

D. Variation

A. For each of the following, state whether it is an example of direct variation or not an example of direct variation.

1. The velocity of a body falling from rest varies directly with the time of all.
2. The distance a body falls from rest is directly proportional to the square of the time of fall in seconds.
3. The volume of a sphere varies directly as the cube of the radius.
4. The volume of a given mass of gas varies inversely with the pressure.

5.

X	Y
4	12
5	15
6	18
7	21

6.

X	Y
1	5
2	6
3	7
4	8

7.

X	Y
5	20
7	28
9	36
11	44

8.

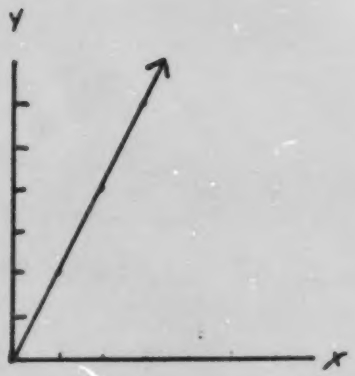
X	Y
0	1
1	2
2	5
3	10

9.

X	Y
0	0
1	2
2	8
3	18

10. $b = 3a$
11. $y = 2x^3$
12. $y = x^2 + 4$
13. $d = M/3$

14.



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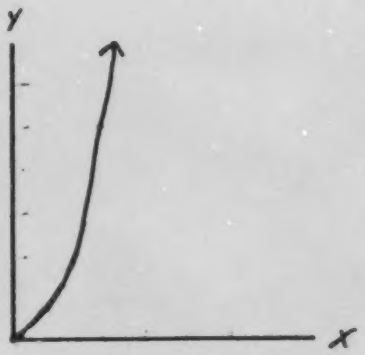
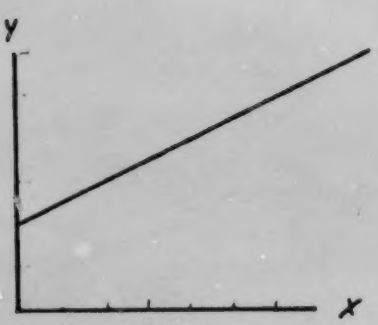
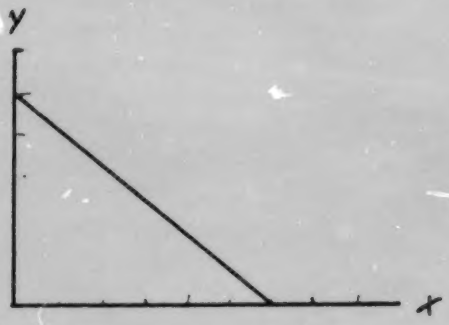


16. 

17. 

A-2 For each of the following, determine the constant of variation.

1. A deisel truck burned 160 L of fuel while travelling 1000 km.
2. A team purchased 6 volleyballs for $240.
3. Eight new Math texts cost $96.
4. y varies directly with x. Y is 16 when x is 4.
5. $d = \frac{5}{2}c$
6. $m = an$
7. b varies directly with the square of a. b is 20 when a is 2.
8. Twenty meals at a banquet cost $90.
9. d varies directly as c. d is 6 when c is 12.
10. $150 purchases 50 golf balls.

A-3 Solve the following problems:

1. A small engine burned 4 litres of fuel in 2 hours. How many litres would it burn in 10 hours?
2. An elevator rose 150 m in 2 minutes. How long would it take to rise 450 m?
3. y varies directly as x. y is 10 when x is 4. Find y when x = 6.
4. a varies directly as the square of b. a is 18 when b is 3. Find a when b is 5.
5. The mass of a metal sphere varies directly as the cube of the diameter. A sphere with d = 6 cm has a mass of 120 g. What is the mass when d = 2 cm?
6. The distance through which an object falls from rest varies directly as the square of its time. An object falls 45 m in 3 seconds. How far does it fall in 5 seconds?
7. The time in seconds for one swing of a pendulum varies directly as the square root of the length in centimetres. If a 100 cm pendulum swings once in 2 seconds, what time is required for a swing of a 64 cm pendulum?

A-4 Direct variations can be expressed in four ways:
a) word statement
b) equation
c) graph
d) chart

For each of the following, express the variation in the other three forms:

1. $y = \frac{3}{2}x^2$

2. $m = 4f$

3. n varies directly with the square root of m

4. q varies directly with the cube of b

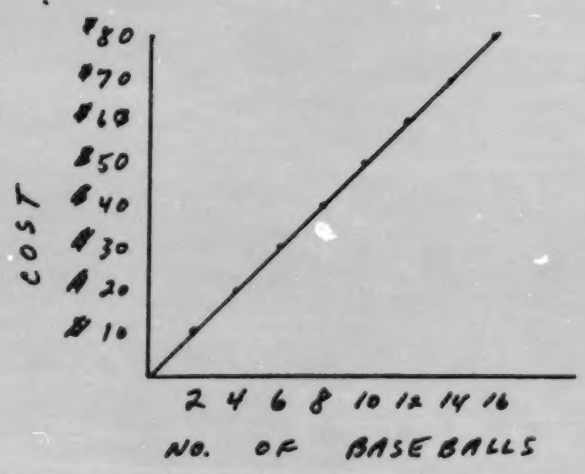
5.

f	t
1	2
2	8
3	18
4	32

6.

x	y
0	0
1	2.5
2	5
3	7.5

7.



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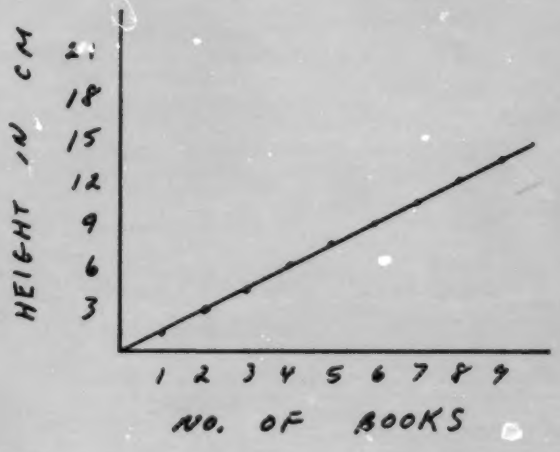


B-1 Identify those problems which involve partial variations.

1. The cost of printing sale bills 100 cm by 40 cm is $70 plus $20 per hundred posters.

2. The distance a hiker travels depends on the rate of speed and the time.

3. The rent of an arena used for hockey games is $100 plus 20% of the ticket revenue.

4. $p = 40 + 3q$

5. $y = 4x^5$

6. $m = 10 + .3n$

7. $y = 2x^2 + 4$

8.

No. of passengers	Cost
0	200
10	300
20	400
30	500
40	600

9.

x	y
0	0
1	1
2	2
3	4.5
4	8

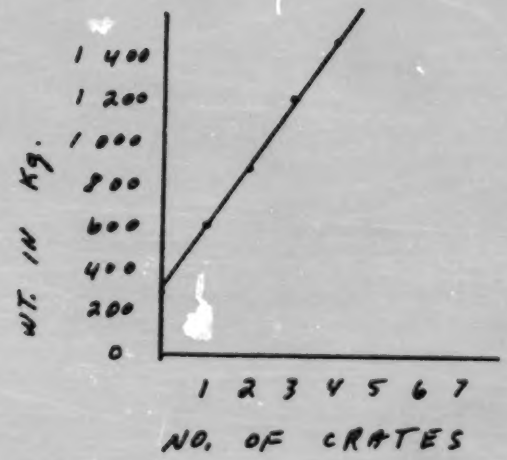
10.

a	b
1	12
2	14
3	16
4	18
5	20

11.

c	d
0	3
1	4
2	7
3	12
4	19

12



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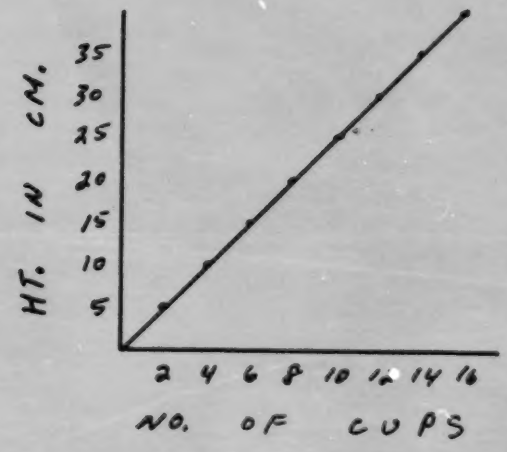

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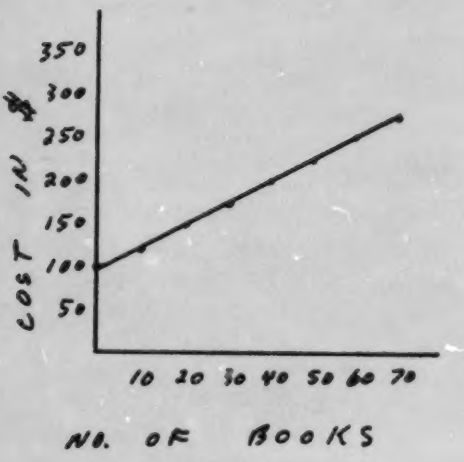


B-2,3 For each of the following, determine the constant of variation and solve the problem.

1. The cost of taking a group on an excursion was quoted at $300 if 10 people went and $600 for 40 people. What would it cost if 25 people decided to go? (The total cost is based on a fixed cost plus so much per person.)

2. The weight of the load for a truck hauling crates of parts depends upon the weight of the truck and the number of crates carried (each crate weighs same amount). A truck carrying 6 crates weighs 5000 kg while a truck carrying 10 crates weighs 7000 kg. What would the truck weigh if it carried 11 crates?

3. Find y if x = 14

x	y
2	12
4	18
6	24
8	30

4. Find d if q = 2

q	d
1	22
3	38
5	70
6	92

5. The cost of a trip to play basketball is based on a fixed cost plus a given amount per person going. The cost for 18 people is $85.50, and the cost for 26 people is $103.50. What would be the cost for 29 people?

6. The cost of sowing a crop is determing by a fixed cost plus a given amount per hectare sowed. 200 hectares cost $23 000 to sow and 400 hectares cost $45 000.
   a) What would it cost to sow 270 hectares?
   b) How many hectares could be sowed for $34 000?

7. The cost of butchering hogs is determined by a fixed fee plus an additional fee for each hog. 20 hogs cost $600 to butcher and 70 hogs cost $1600.
   a) What would it cost to butcher 38 hogs?
   b) How many hogs were butchered if the cost was $1060.

C-1 Identify those problems which involve inverse variation.

1. The number of meals one can purchase with a given sum of money varies inversely as the cost of each meal.

2. $y = \frac{k}{x}$

3. $y = 2x$

4. The total cost of cleats for a baseball team is determined by the number of pairs of cleats needed.

5. The time required to drive a given distance varies inversely with the rate of speed.

6. $L = \frac{144}{d^2}$

7. $y = \frac{a^2}{4}$

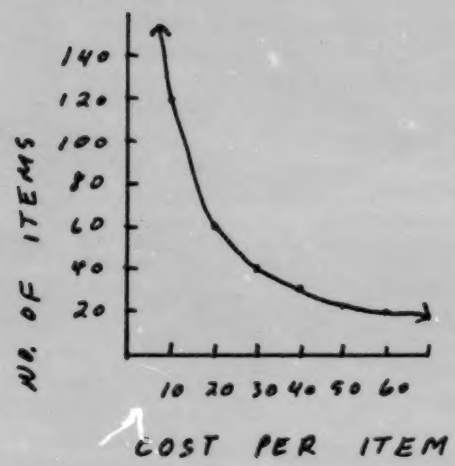
8.

a	b
10	120
20	60
30	40
40	30

9.

q	y
6	50
10	30
12	25
15	20

10.



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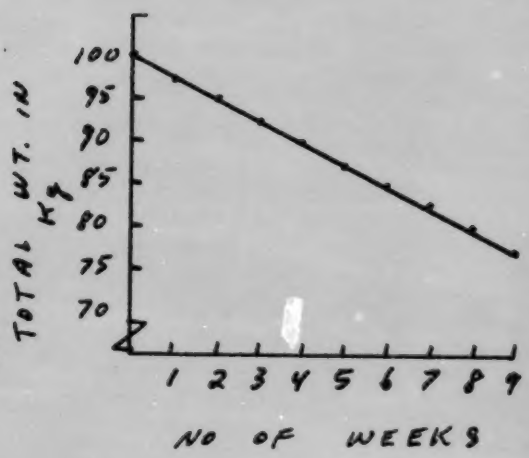


C-2,3 For each of the following determine the constant of variation and solve the problem.

1. The time, in hours, required to cover a given distance varies inversely as the rate of speed. If a motorist took 7 hours to cover the distance when driving at a rate of 80 km/hr., how long will it take if he drives at a rate of 100 km/hr.?

2. Given: m varies inversely with d
   $m = 10$ if $d = 30$

   a) Find the constant of variation.
   b) Find d if $m = 20$.
   c) Find m if $d = 30$.

3. The voltage in a circuit varies inversely with the resistance. In a circuit of given voltage, the current is 2 amps when the resistance is 3 ohms. What is the resistance if the current is 6 amps?

4. The number of meals one can purchase at a banquet varies inversely as the cost of the meal. If 200 meals cost $6 each, what would be the cost of each meal if 150 meals were ordered?

5. The time required to complete a construction project is inversely proportional to the number of men working. If 24 men take 30 days to do the job, how long would 36 men take?

6. m varies inversely with n, and m = 25 when n = 9. What is the value of m when n = 15?

7. The amount a rectangular beam of fixed length bends varies inversely with the cube of its depth. A 50 mm by 100 mm (2 x 4) stud 4 m long sags 12.7 mm under a given load. How much would a 50 mm x 150 mm (2 x 6) stud 4 m long bend under the same load?

D-1 For each of the following identify each as a direct, partial, or inverse variation and solve the problem.

1. The number of blows one receives is dependent upon the number of bad jokes one tells. If a joker received 6 blows for 2 bad jokes, how many blows would he receive for 3 bad jokes?

2. The number of spherical shot needed to fill a box varies with the cube of the radius of the shot. When r = 3 mm, n = 1000 shot. How many shot can be held if the radius is 5 mm?

3. The cost of holding a rally for a basketball game is a fixed sum plus an additional sum for each person attending. If $120 was charged when 100 people attended and $137.50 for 125 people, what would the charge have been for 150 people?

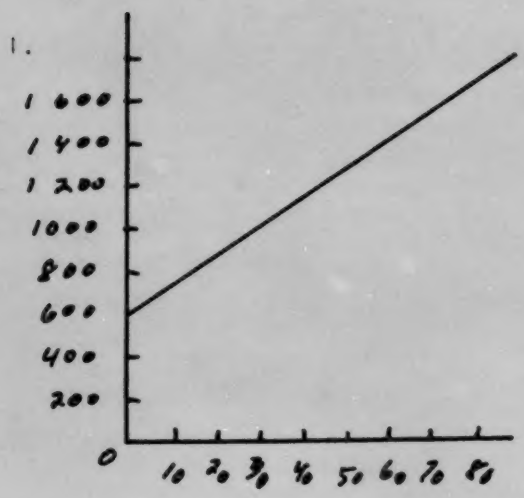
4. If a Canadian dollar is worth $0.785 U.S., what would it cost to purchase $1,000 U.S. dollars?

5. The romantic attraction between compatible males and females varies inversely with the square of the distance between them. If the strength of the attraction is 1600 loving at 4 m, what would the strength of the attraction be at .5 m?

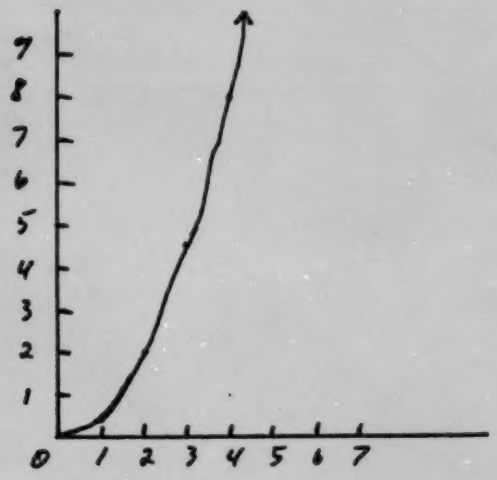
6. Cab fare is determined by adding so many cents per km to a fixed charge. If a ride of 4 km cost $3.25 and one of 8 km cost $5.50, what would a ride of 5 km cost?

D-2 For each of the following, identify the type of variation.

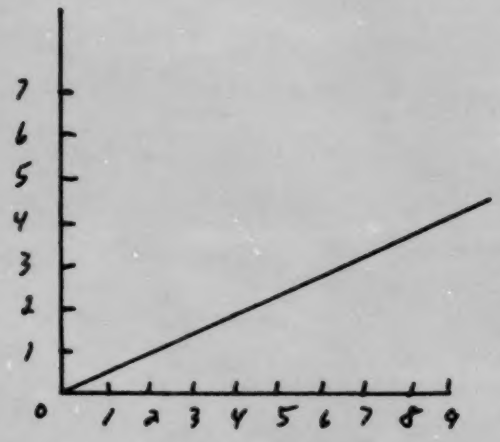
1.


1 600
1 400
1 200
1000
800
600
400
200
0
10 20 30 40 50 60 70 80

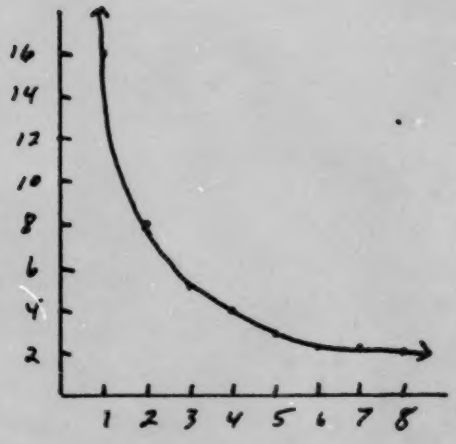
2.


8
7
6
5
4
3
2
1
0
1 2 3 4 5 6 7

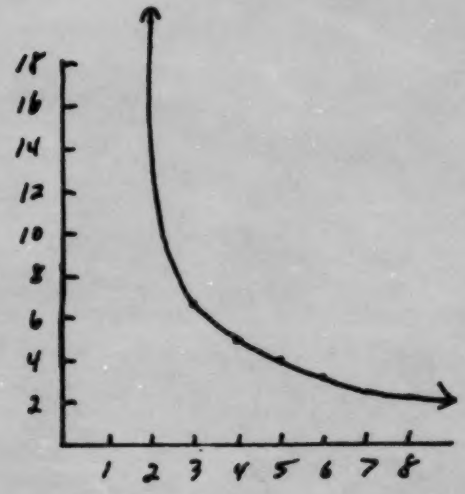
3.


7
6
5
4
3
2
1
0
1 2 3 4 5 6 7 8 9

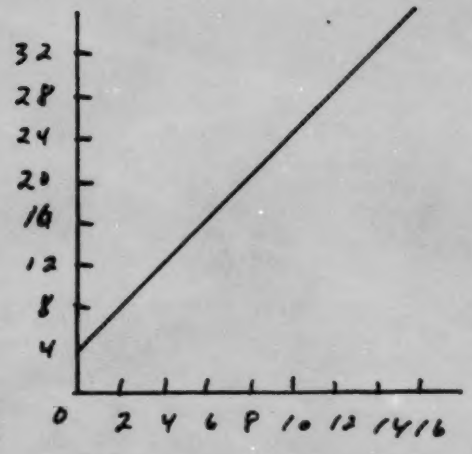
4.


16
14
12
10
8
6
4
2
1 2 3 4 5 6 7 8

5.


18
16
14
12
10
8
6
4
2
1 2 3 4 5 6 7 8

6.


32
28
24
20
16
12
8
4
0
2 4 6 8 10 12 14 16

Answer Key

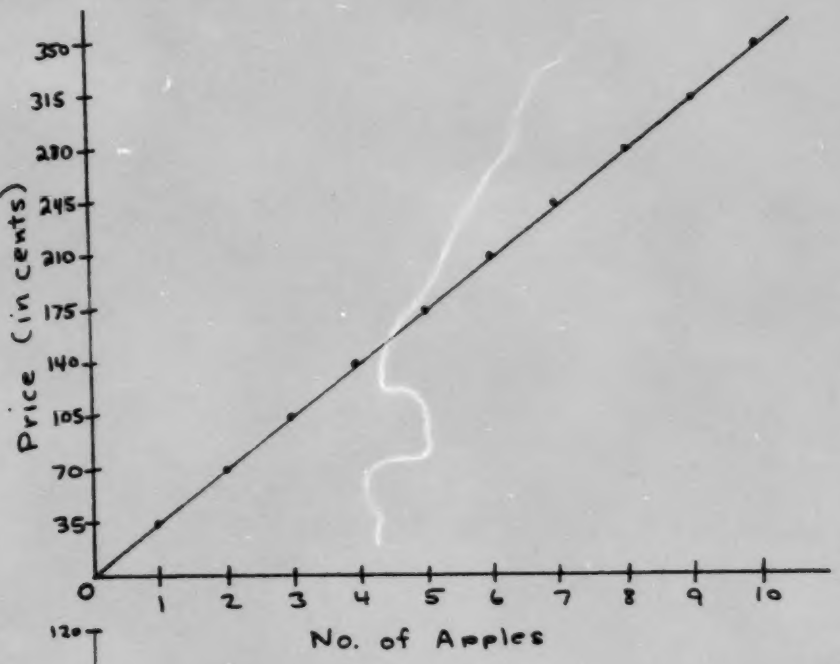
A Graphing

NOTE: The slope of the graphs will be affected by the scale selected.

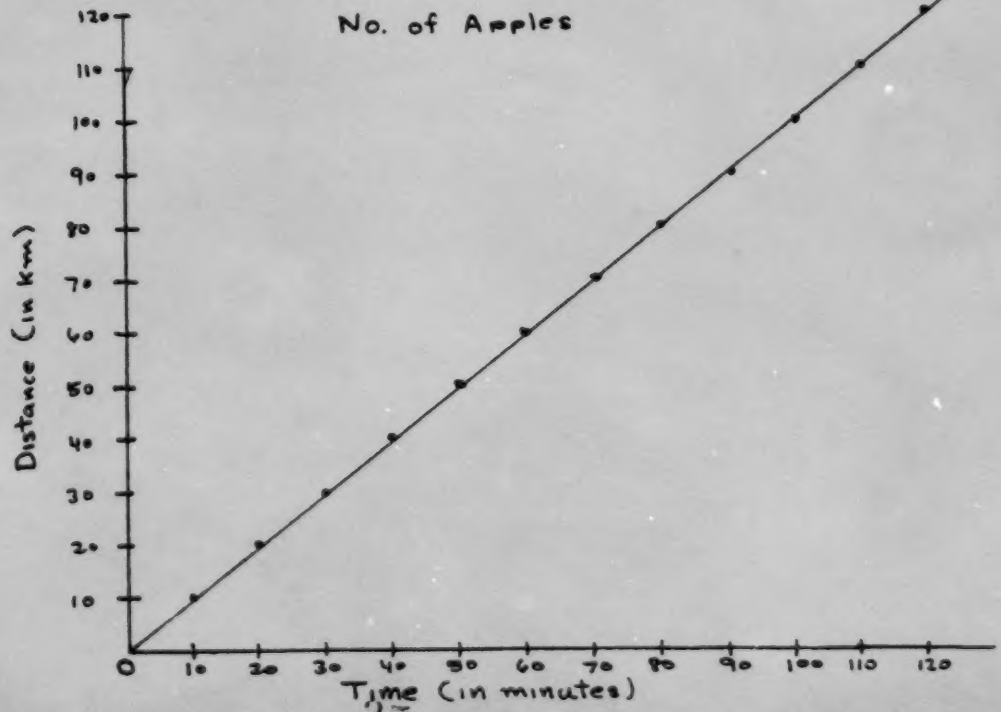
A-1 1-3 Answers will vary according to the data collected.

A-2 1. Answers will vary.

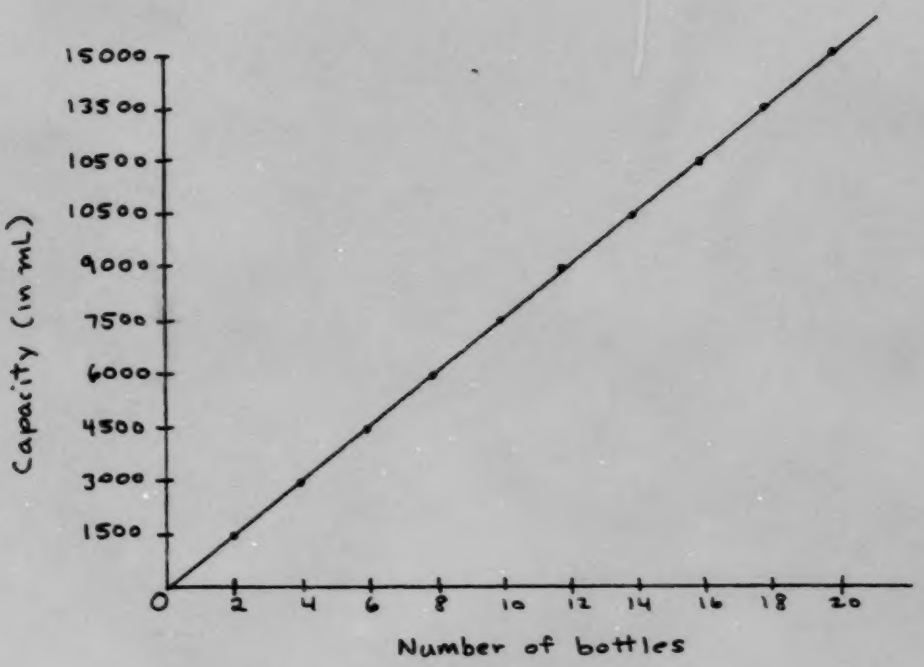
2. a)



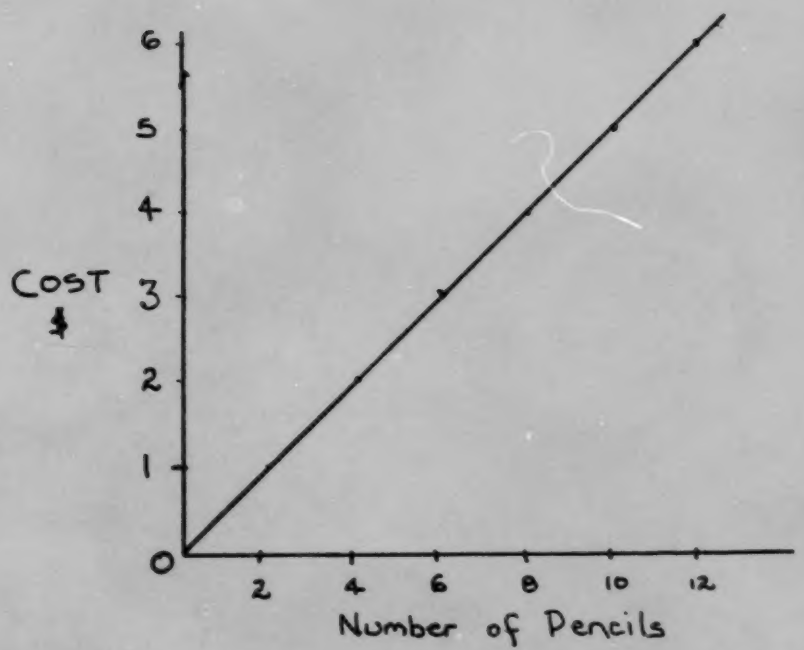
b)



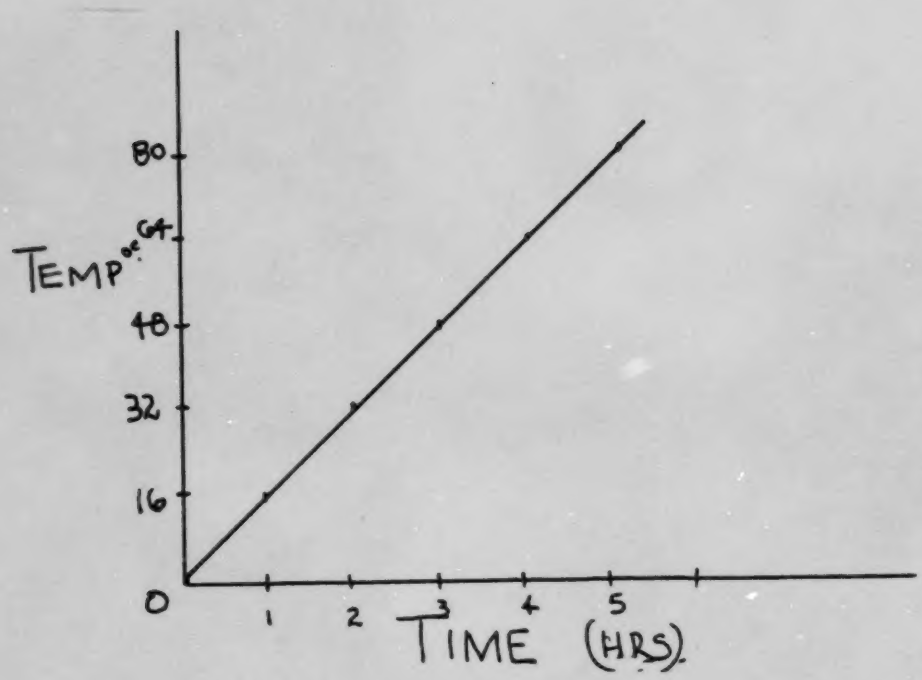
c)


15000
13500
10500
10500
9000
7500
6000
4500
3000
1500
Capacity (in mL)
0
2
4
6
8
10
12
14
16
18
20
Number of bottles

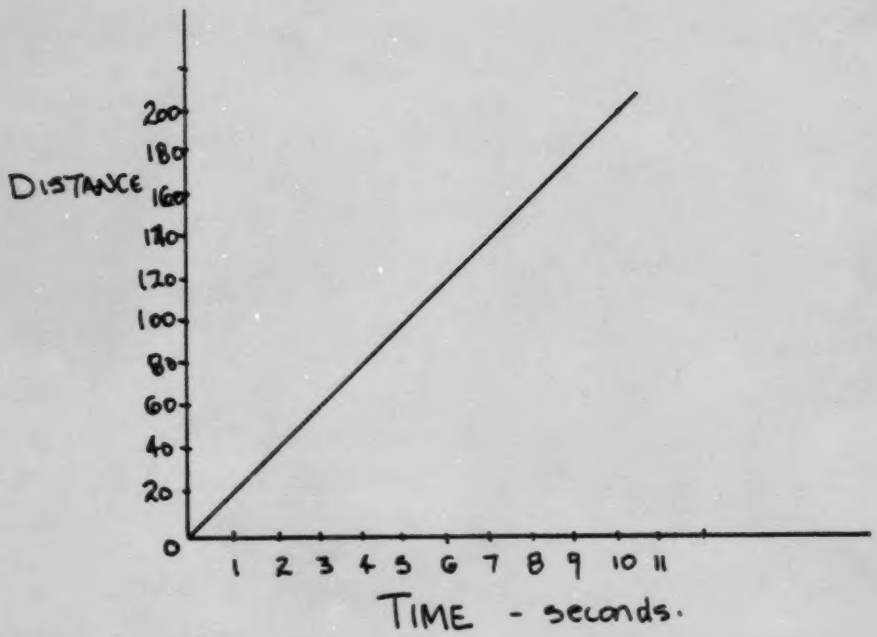
3. a)


6
5
4
COST
$
3
2
1
0
2
4
6
8
10
12
Number of Pencils

b)


80
64
48
32
16
0
TEMP °F
1
2
3
4
5
TIME (HRS).

c)


200
180
160
140
120
100
80
60
40
20
0
DISTANCE
1 2 3 4 5 6 7 8 9 10 11
TIME - seconds.

A-4

1. a) 15 km
   b) 93 km
   c) 150 km

2. a) 5250 ml
   b) 11 250 ml
   c) 18 000 ml
   d) 15 750 ml

3. a) 12°C
   b) 36°C
   c) 88°C

4. a) 150 m
   b) 220 m

5. a) $10
   b) $17.49
   c) $11.66 A good estimate is between $11 and $12
   d) $33.32 A good estimate is $32 to $35
   e) $59.98

A-5

1. $W = N \times M$ where W = total on pan
   N = number of marbles
   M = mass of 1 marble

   $H = N \times T$ where H = height of pile
   N = number of texts
   T = height of 1 text (thickness)

2. a) $C = N \times A$ where C = total cost
   N = number of apples
   A = cost of 1 apple

   b) $D = V \times T$ where D = distance travelled
   V = velocity (uniform)
   T = time

   c) $V = N \times C$ where V = total volume
   N = number of bottles
   C = capacity of 1 bottle

3. a) $C = N \times P$ where C = total cost
   or $C = N50$ N = number of pencils
   P = price of 1 pencil (50¢)

b) $T = C \times I$ or $T = 8 \cdot I$ where T = temperature, C = temperature increase of I interval, I = number of intervals

c) $D = V \times T$ or $D = 20 \cdot T$ where D = distance travelled, V = velocity of object, T = time (seconds)

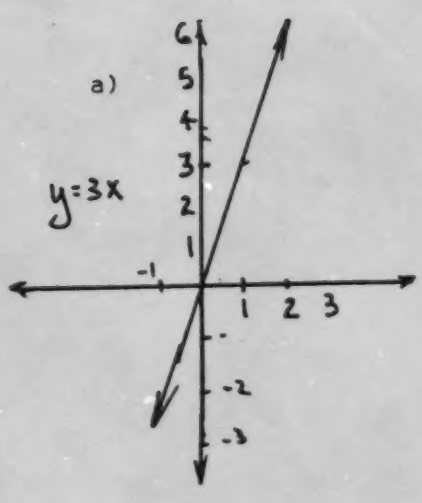
4. $C = N \times P$ or $C = N \cdot 16$ or $C = 16N$ where C = total cost, N = number of footballs, P = price of 1 ball

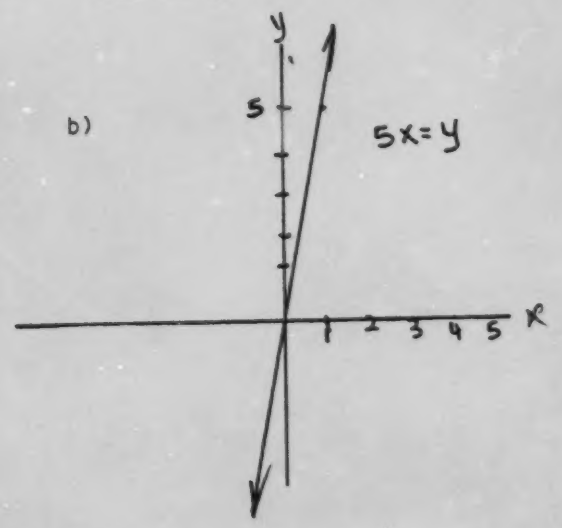
5. $D = V \times T$ or $D = 2 \times T$ or $D = 2T$ where D = distance in kilometres, V = velocity - constant, T = time - minutes

6. $M = N \times W$ or $M = N \times .5$ or $M = .5N$ where M = total mass, N = number of boxes, W = mass of 1 box - constant

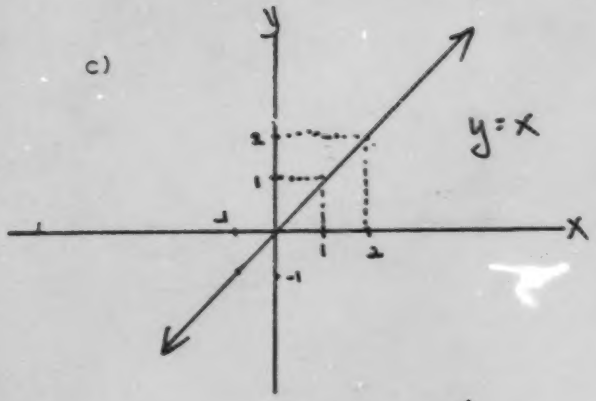
NOTE: For answers for A-5, students need not have the same letters, but should indicate what their chosen letters represent.

A-6 1.

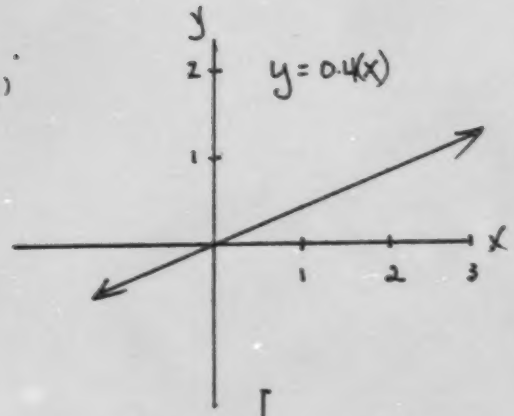




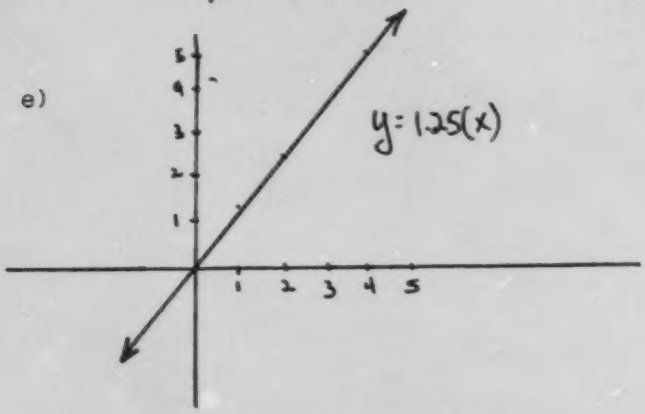
c)


y
x
y = x
2
1
-1
1
2

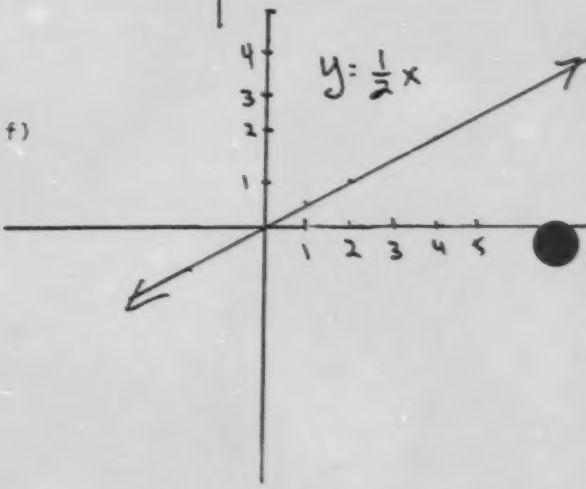
d)


y
x
y = 0.4(x)
2
1
1
2
3

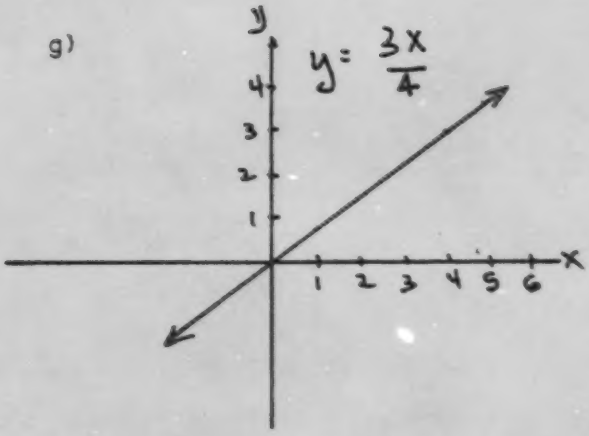
e)


y = 1.25(x)
5
4
3
2
1
1
2
3
4
5

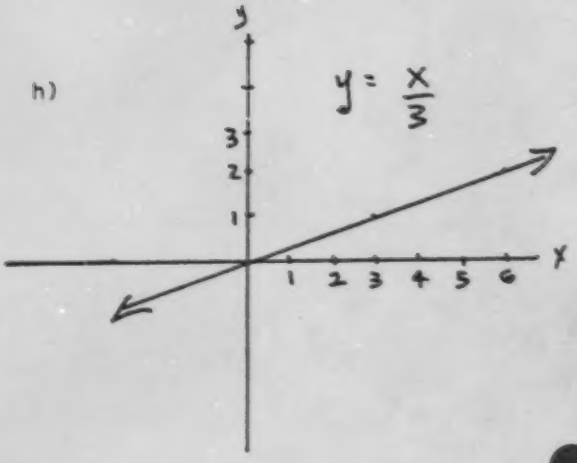
f)


y = 1/2 x
4
3
2
1
1
2
3
4
5

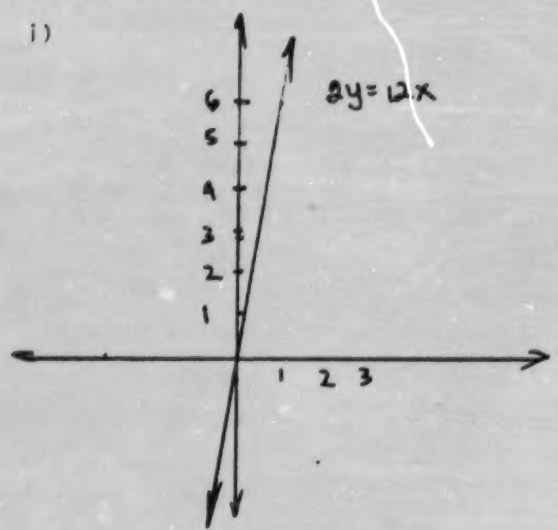
g)


y
x
y = 3x/4
4
3
2
1
1
2
3
4
5
6

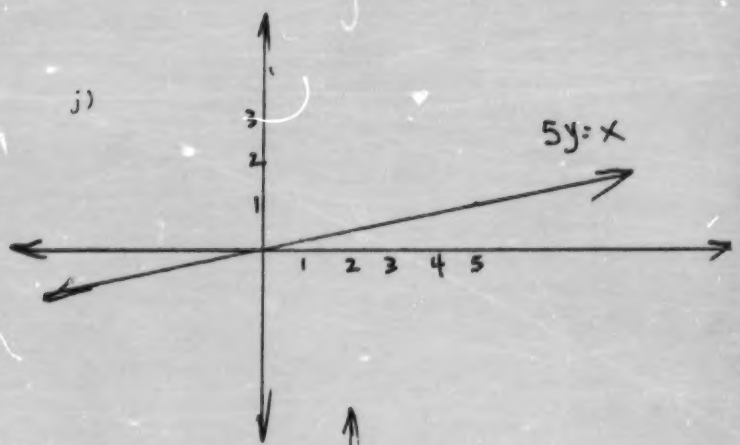
h)


y
x
y = x/3
3
2
1
1
2
3
4
5
6

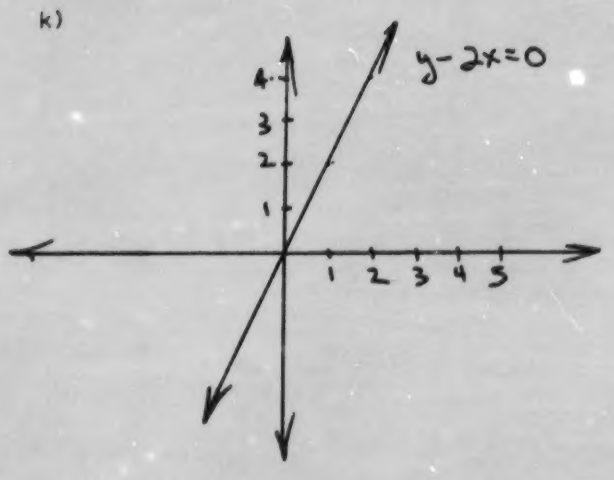
i)



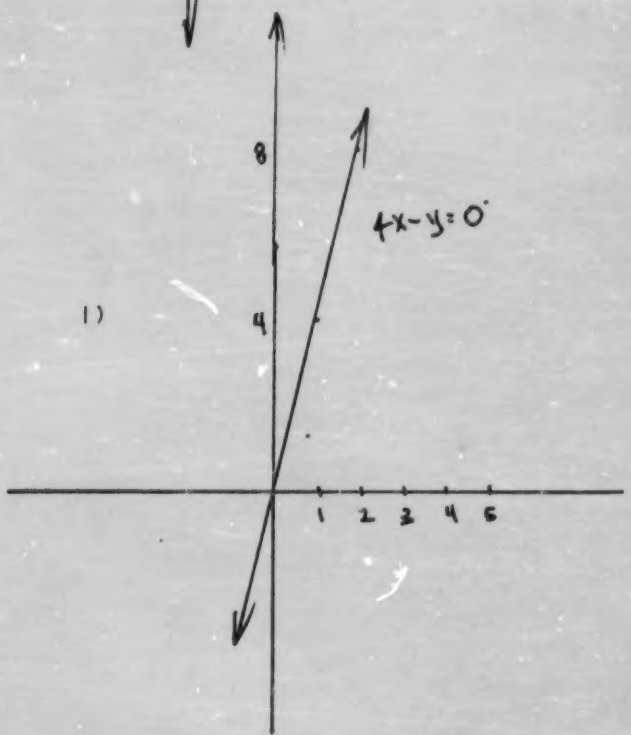
j)



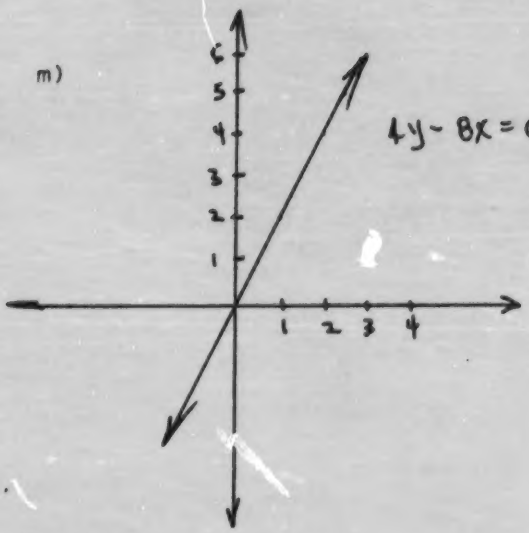
k)



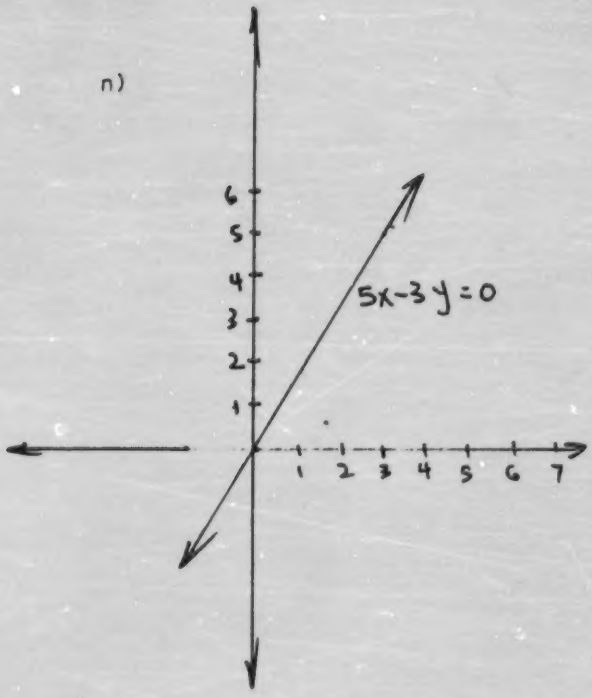
l)



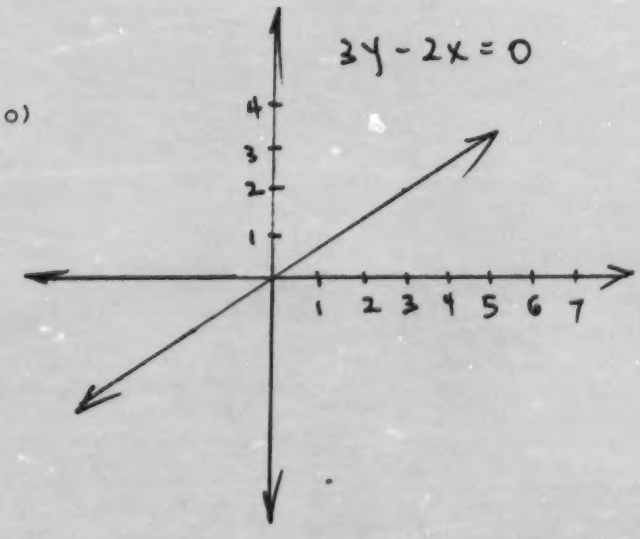
m)



n)

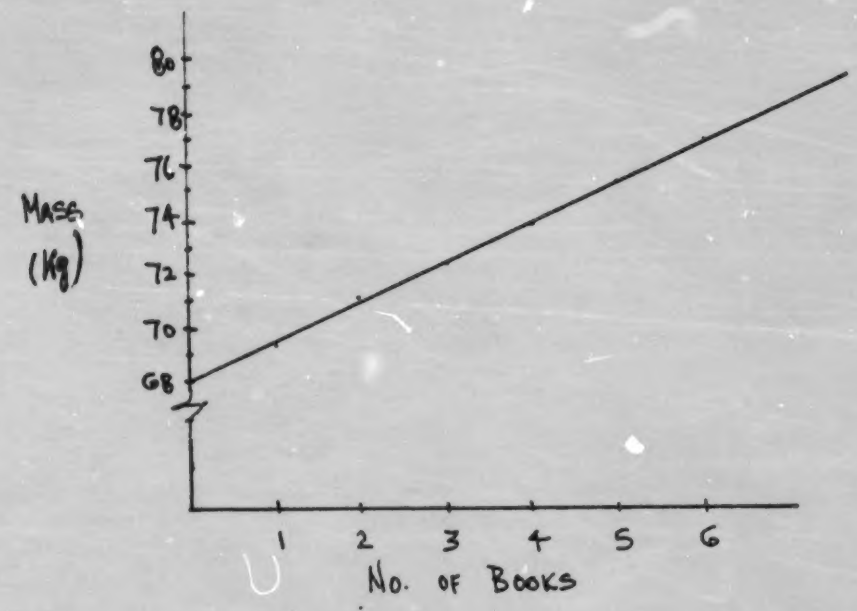


o)

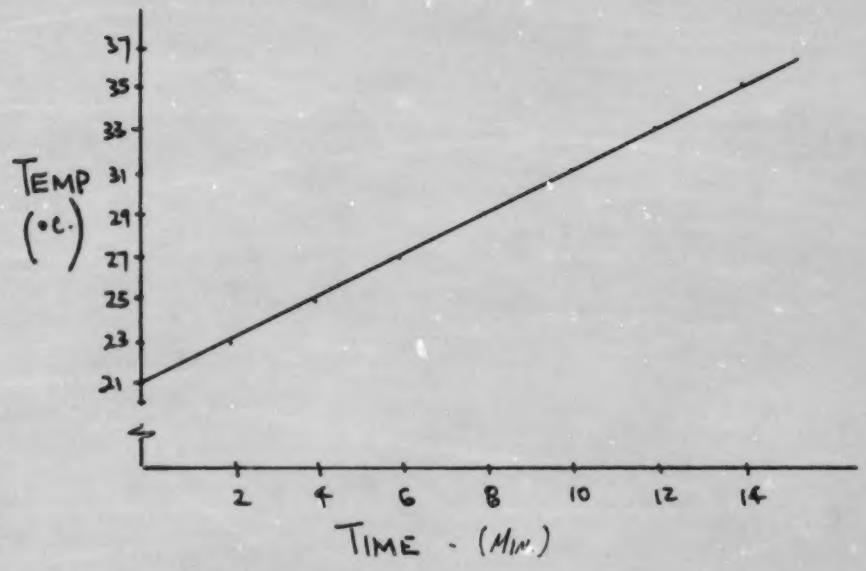


B Partial Linear Relations

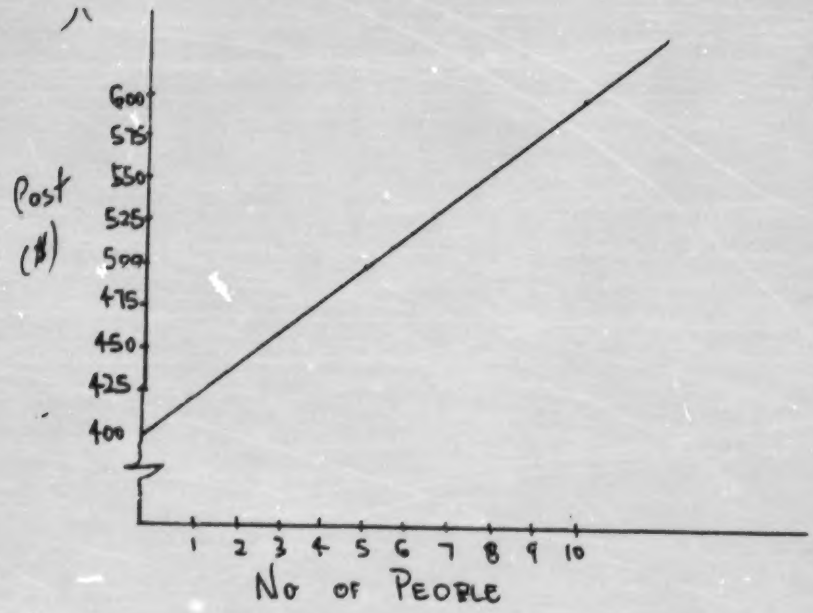
B-2 a)



b)



c)



B-4 a) i) 1.5 kg
ii) 86 kg

b) i) 26°C
ii) 30.5°C
iii) 41°C
iv) 1°C

c) i) $640
ii) $70
iii) $86.67

B-5 a) W = 68 + n(1.5) or W = 1.5N + 68 where W is weight in kg and n is the number of books; c = 400 + n(20)

b) C = 45 + n(2.95). Estimate, about $22.10

B-6 a) 1. $4y = 3x - 10$
$y = 3x - 10/4$
$y = 3/4x - 10/4$ or $y = 0.75X = 2.5$

2. $2y + 4x = 8$
$2y = -4x + 8$
$y = 2x + 4$

3. $6x - y = 3$
$-y = -6x + 3$
$y = 6x - 3$

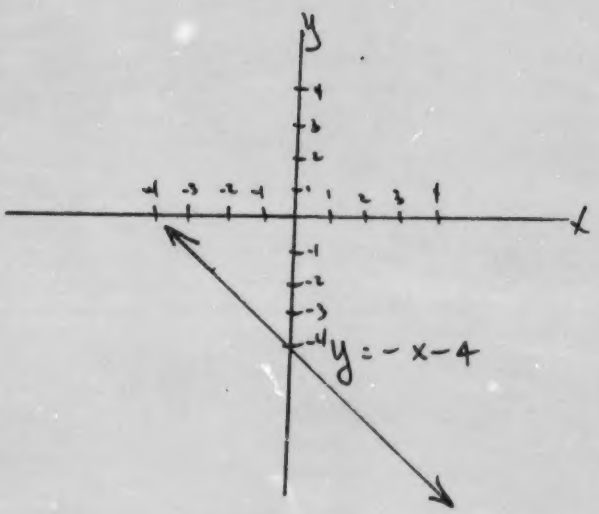
4. $8x + 2y + 2 = 0$
$2y = -8x - 2$
$y = -4x - 1$

b) 1. $y = \frac{2}{3}x - 1$

2. $y = 4x + 2$

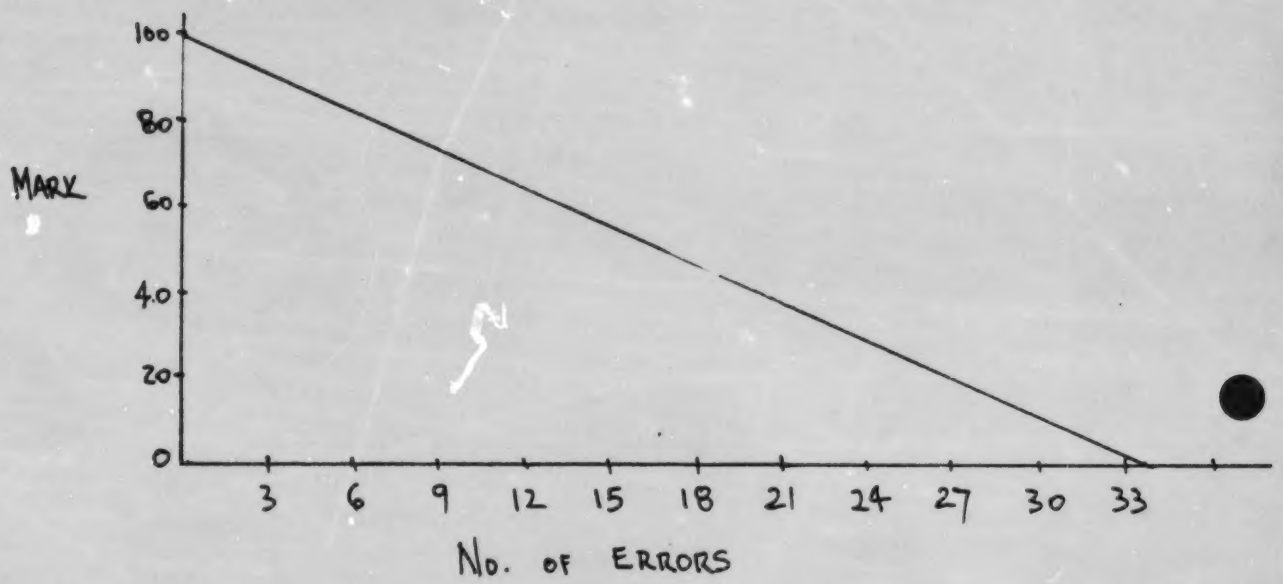
3. a) $y = -x + 4$
b) $y = -x$

The lines are parallel.



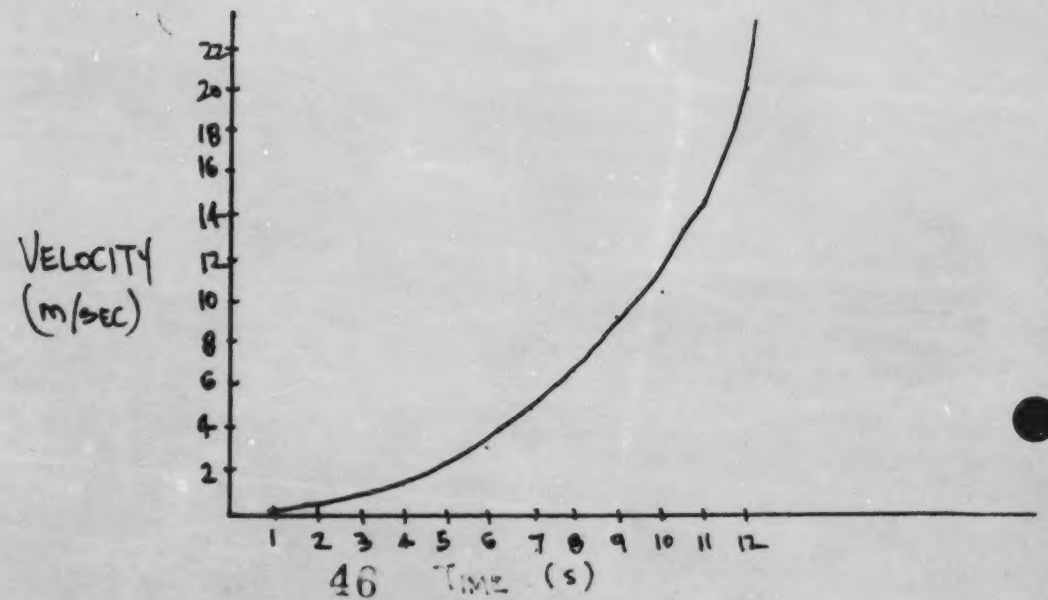
B-7 1. a) $y = -2x - 5$
b) $y = \frac{4}{3}x - 3$
c) $y = \frac{9}{4}x + 4$
d) $y = \frac{-5}{2}x + 17$

2. a) C = 12(n) + 100
   b) C = .08(km) + 28 - for 650 km, cost is .08(650) + 28 = $80
                      - for 1000 km, cost is .08(1000) + 28 = $108
   c) Equation is: Mark = -3 (# of errors) + 100
                   M = -3(E) + 100
      Minimum errors for failure: 49 = -3(E) + 100
                                  -51 = -3(E)
                                  E = 17

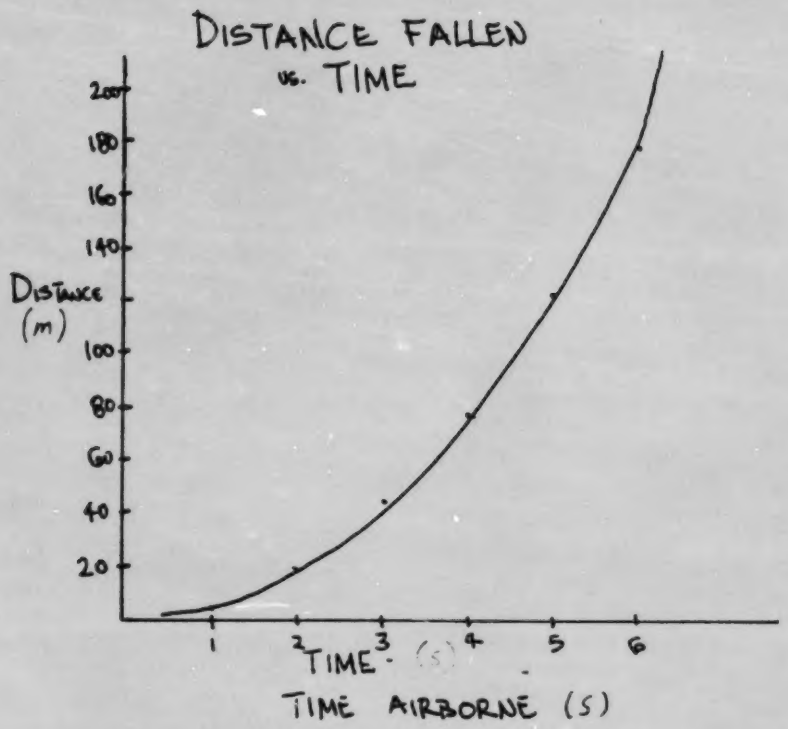


C Non-Linear Relations

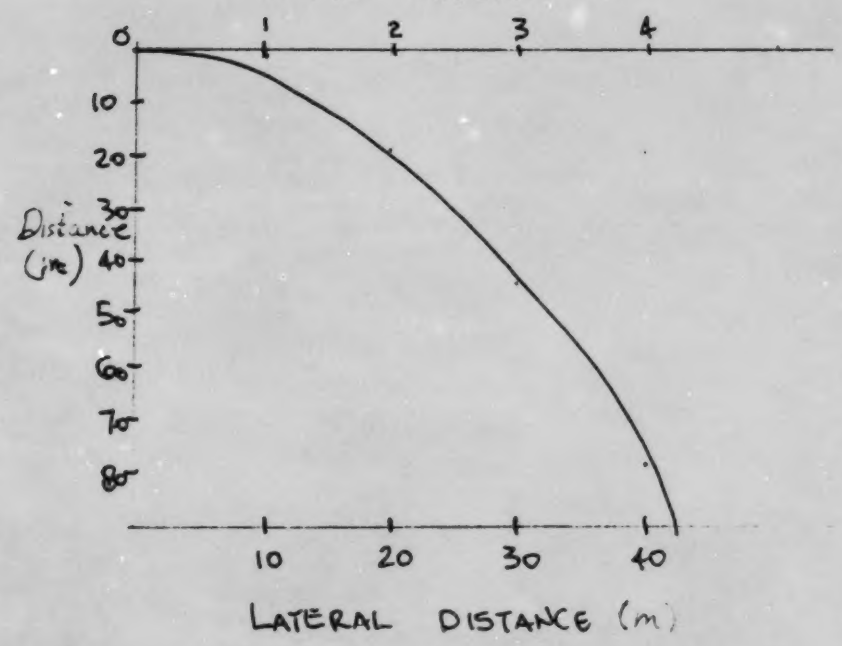
1. a)



b)



c)



NOTE: This form of graph (c) is more acceptable than (b) as it visually portrays the actual path of the object.

2. a) 11.2 seconds, 13 seconds (A good estimate is acceptable)
   b) (Estimate: 240-250 m), 400-500 m
   c) 80 m

B.1

1. a) 16
   b) 125
   c) 64
   d) 1
   e) 3
   f) 16
   g) -27
   h) 16

2. a) 3^5
   b) 4^{11}
   c) 2^{13}
   d) 7^{13}
   e) 5^{11}
   f) 2^{12}
   g) 3^{12}
   h) t^9
   i) x^8
   j) m^{11}
   k) t^9
   l) b^{18}

3. a) 36
   b) 100
   c) 1
   d) 343
   e) 32
   f) 9
   g) 5
   h) 9
   i) x^4
   j) a^7
   k) n^4
   l) w^{10}

4. a) 1/8
   b) 1/9
   c) 1/64
   d) 1/64
   e) 1/6
   f) 1/128
   g) 1/256
   h) 1/729

5. a) 9
   b) 1/2
   c) 4
   d) 1/4
   e) 64
   f) 2
   g) 6561
   h) 2
   i) 1/3
   j) 1/16
   k) 1/32
   l) 1/1000
   m) 6561
   n) 4096
   o) 6
   p) 1/3

B.2

1. a) 13a
   b) 16x
   c) 7m
   d) -7t
   e) 7x + 3
   f) 36 - 26x
   g) $a^2 - 5a - 1$
   h) 2x - 12
   i) 1 + 15x
   j) 8x + 15
   k) 9x - 13
   l) 2x - 5
   m) 9x - 1
   n) $a^2 + 5a - 5$

2. a) 14x
   b) $12x^3$
   c) $-20a^4$
   d) $48b^5$
   e) $-21m^4$
   f) $6x^5$
   g) $-30a^8$
   h) $56m^{12}$
   i) $12x + 20$
   j) $30x - 42$
   k) $-21m^2 - 28m$
   l) $24t^4 + 12t^3$
   m) $x^2 + 8x + 15$
   n) $a^2 + a - 30$
   o) $t^2 + 14t + 4q$
   p) $66^2 - 8b - 8$
   q) $10m^2 - 17m - 20$
   r) $9x^2 - 25$

3. a) 11a + 2
   b) -66 + 50
   c) 3m + 5
   d) -44m + 12
   e) $-2t^2 + qt$
   f) $5x^3$
   g) $2m^3$
   h) $2z^6$
   i) $8m^3 + 2m$
   j) $2w^3 + 3w^2$
   k) $9m^6 - 14m^3 + 23$

4. a) $-5x - 3$
   b) $12x^2 + 20x + 8$
   c) $21a - 6$
   d) $15m + 4$
   e) 16
   f) 15
   g) $6x^2 - 10$
   h) $x + 3$
   i) $a^2 - 2a - 35$
   j) $16x + 8$
   k) $9x^2 - 12x - 5$

B.3
a) 3.6×10^4
b) 7.3×10^3
c) 8.2×10^7
d) 1.25×10^9
e) 8.3×10^7
f) 2.3×10^{10}
g) 1×10^{20}
h) 1×10^{12}
i) 8.1×10^5
j) 2×10^{14}

B.4
a) 3.4×10^{-3}
b) 5.6×10^{-4}
c) 8×10^{-8}
d) 9.4×10^{-3}
e) 7.05×10^{-7}
f) 9.58×10^{-1}
g) 6.753×10^{-6}
h) 1.3×10^{-3}
i) 6.25×10^{-4}
j) 7.36×10^{-14}

B.5
a) 34 600 000
b) 0.001 15
c) 86 000
d) 0.000 003 04
e) 0.000 888
f) 100 900 000
g) 450 000
h) 0.000 000 033
i) 1 000 000
j) 7 820 000

C.1 1. a) 5
b) 9
c) 32
d) -7
e) -15
f) 5
g) 10.5 (21/2)
h) 17.3 (52/3)
i) 4
j) 4
k) 9
l) 10
m) -5.3 $(-\frac{16}{3})$
n) 1
o) -3.3 $(-\frac{10}{3})$
p) -1
q) -3
r) 0
s) -132
t) -5
u) -3
v) -21
w) -1.71 $(-\frac{12}{7})$
x) -1.75 $(-\frac{7}{4})$
y) 21
z) Ø

2. a) -2
b) 4
c) 1
d) -35
e) 16
f) 84
g) 31
h) 1.73 (57/33)
i) -0.2 $(-\frac{1}{5})$

3. a) 81
   b) 10
   c) 3
   d) -2.6(-8/3)
   e) -14
   f) -0.5 (1/2)
   g) 0.285 (2/7)
   h) 0.83 (5/6)
   i) -20
   j) 4
   k) 3
   l) -48
   m) -0.285 (-2/7)
   n) -7
   o) -16
   p) 1
   q) 0.538 (7/13)

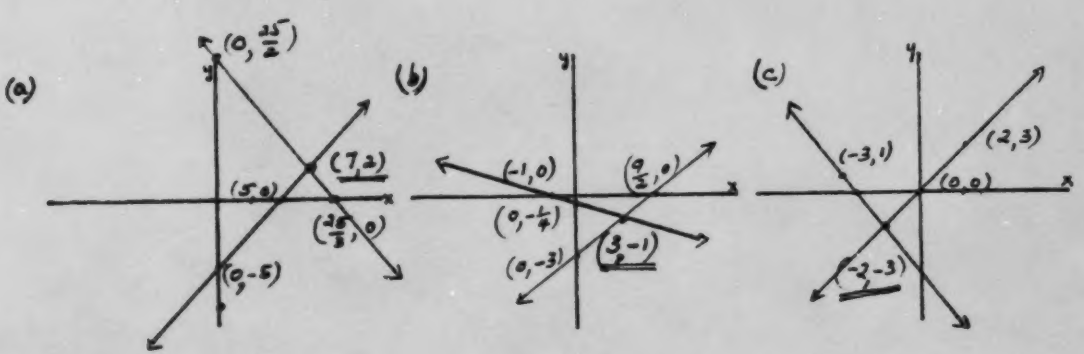
4. a) 31
   b) 2
   c) -1.45 (-48/33)
   d) 15
   e) 9
   f) 3.43 (79/23)
   g) 1.91 (82/43)
   h) 0.3 (3/10)
   i) 0.705

C.2
a) 17,18
b) 7
c) 7
d) 20,21,22
e) J = 32, D = 14
f) 134
g) 17
h) A = 15, P = 20, F = 30
i) 16, 48
j) 77, 79, 81
k) 124, 126, 128
l) 66
m) 5.16 (98/19)
n) n=14, P=28,d=78
o) 2.92 m (38/13)
p) A=81, B=54, C=45
q) 40,16

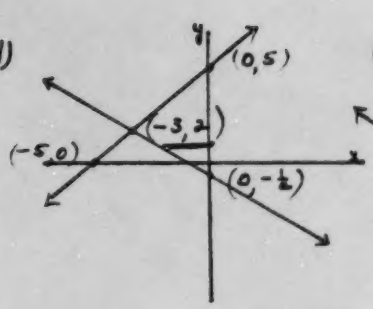
C.3-A
1. Answers will vary.
2. a) (-2, 10/3), (-1, 8/3), (0, 2), (1, 4/3), (2, 2/3)
   b) (-2, 7-7/2), (-1, -11/4), (0, -2), (1, -5/4), (2, -1/2)
   c) (-2, -25/2), (-1, -10), (0, -15/2), (1, -5), (2, -5/2)
   d) (-2, 1) (-1, 0), (0, -1), (1, -2), (2, -3)

C.3-B
a) (4,3) is
b) neither
c) neither
d) (-4,-3) is
e) (7/2, 1) is
f) (-1, 2) is

C.3-C



(d)



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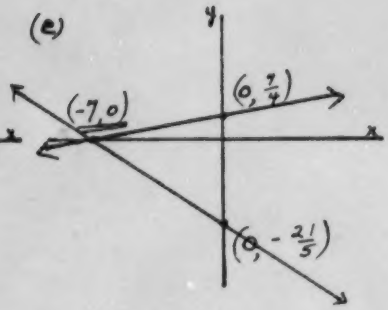

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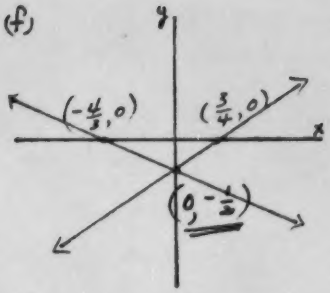
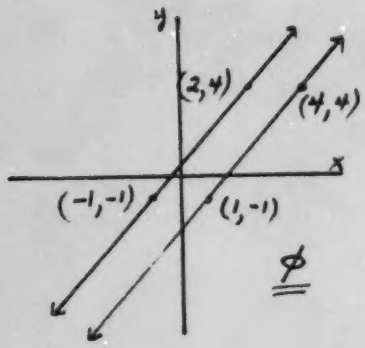


(g)



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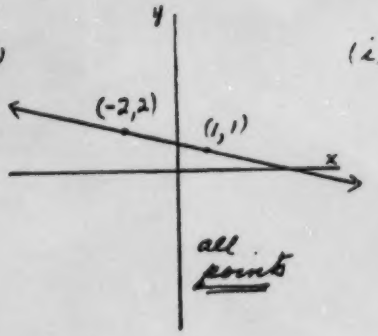
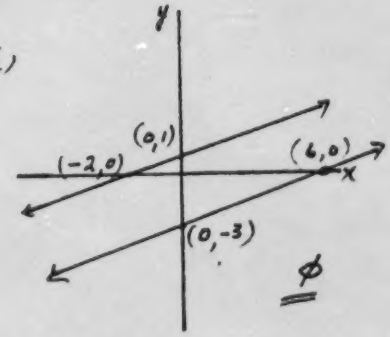
(h)



all points

(i)



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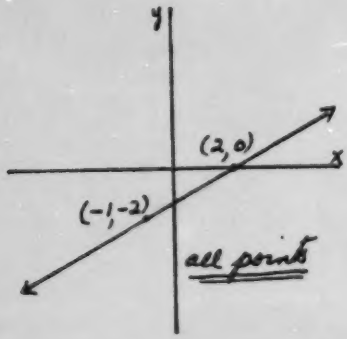
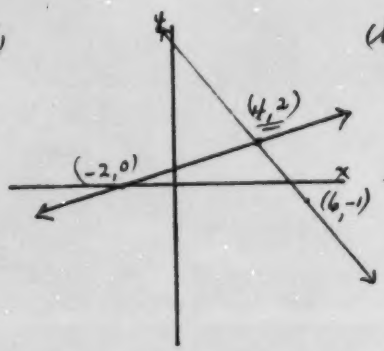
(j)

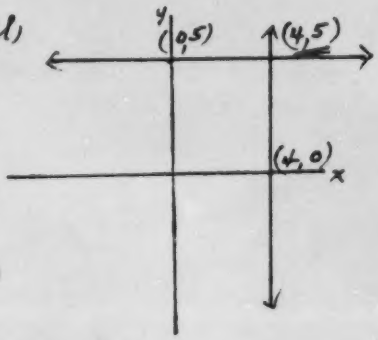


all points

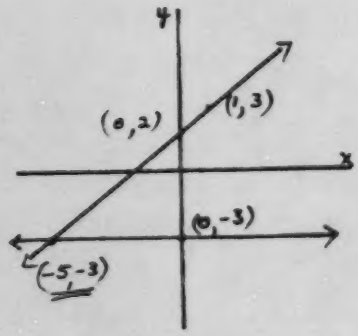
(k)



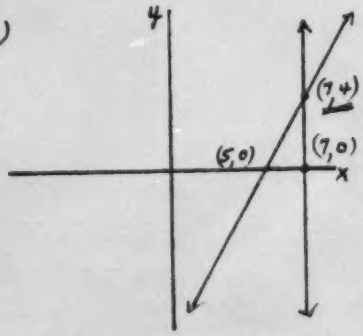
(l)



(m)



(n)



C.3-D

a) (2,3)
b) (1,-3)
c) (2-,-2)
d) (1,0)
e) (10,-5)
f) (0,0)
g) (1/2, 1/3)
h) (1/4, 1/7)
i) (2/3, 1/4)
j) (12,10)
k) (-3,4)
l) (4,0)
m) (4,14)
n) (6,8)
o) (-3,2)
p) (4,5)
q) (4,-6)
r) (7,3)
s) (5,10)
t) (90,100)
u) (24,36)
v) (15,-7)

C.4

a) 18,10
b) 159,91
c) 8,30
d) 28,4
e) $900,$300
f) a=162,s=178
g) p=234,n=58
h) o=165,f=192
i) 85,70
j) $6.80
k) $\sqrt{136}$ (11.66)
l) 6.67 kg of $6.00 candy, 13.33 kg. of $3.75 candy
m) 16 - 60¢, 24 - 30¢
n) 53

D-A-1 A direct variation is a relationship such as $Y = kX^n$ where k and n are greater than zero. The following are examples of direct variation: 1, 2, 3, 5, 7, 9, 10, 11, 13, 41, 15, 17. (6, 8, 12, and 16 are not)

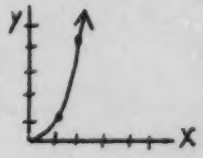
A-2

1. 6.25
2. $40
3. $12
4. 4
5. 2.5
6. a
7. 5
8. $4.50
9. 1/2
10. $3.00

A-3

1. 20 L
2. 6 minutes
3. y = 15
4. a = 50
5. 4.44 g
6. 125 m
7. 1.6 sec.

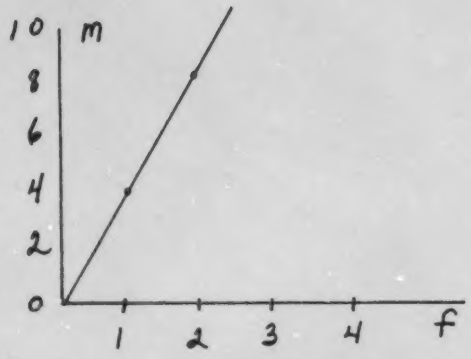
A-4 1. y varies directly as the square of x



x	y
0	0
1	1.5
2	6
3	13.5

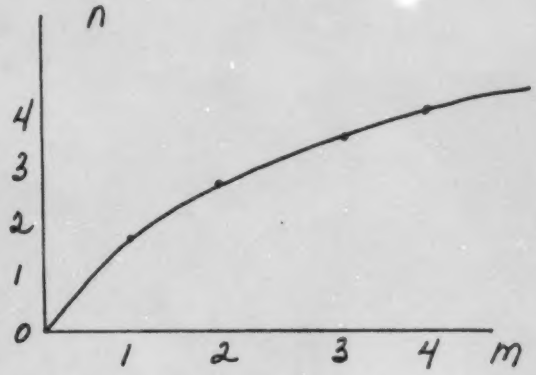
2. $m = 4f$
m varies directly with f

f	m
0	0
1	4
2	8
3	12



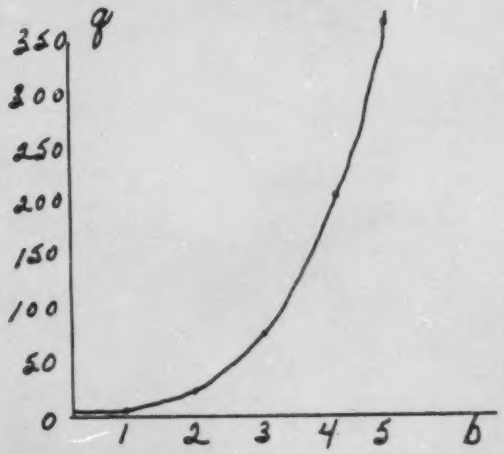
3. If n varies directly with the square root of m, then $n = k\sqrt{m}$
(Note: for the purpose of showing the graph and table of values $k = 2$ is chosen).

m	n
0	0
1	2
2	$2\sqrt{2}$
3	$2\sqrt{3}$
4	4

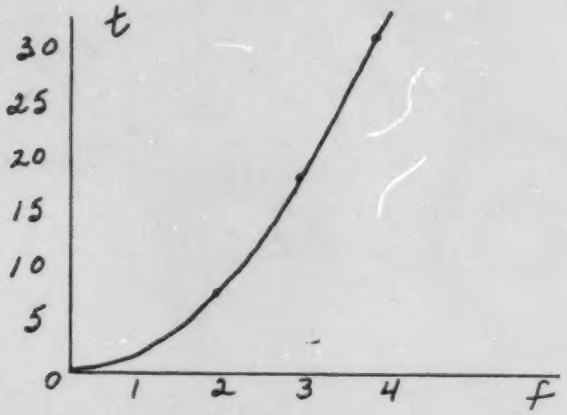


4. $q = kb^3$ ($k = 3$ is chosen to show the chart and graph)

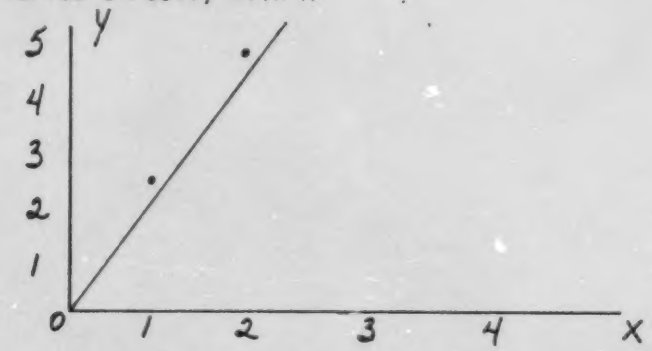
b	q
0	0
1	3
2	24
3	81
4	192
5	375



5. $t = 2f^2$
t varies directly with the square of f.



6. $y = 2.5x$
y varies directly with x



7. The total bill varies directly with the number of baseballs purchased; $C_T = 5B$

B	C_T
2	10
4	20
6	30
8	40
10	50

8. The height varies directly with the number of books; $h = 1.5b$

b	h
1	1.5
2	3
3	4.5
4	6
5	7.5

B-1 1. - 14. The following are examples of partial variation:
1, 3, 4, 6, 7, 8, 10, 11, 12, 14

B-2,3 1. $C_t = \$200 + \$10n$; $(k = 10)$
$C_{25} = 200 + 250 = \$450$

2. $W_t = 500 \times 2000$; $k = 500$ kg
$W_{11} = 5500 + 2000 = 7500$ kg

3. $y = 3x + 6$; $k = 3$; $y_{14} = 48$

4. $d = 20 + 2q^2$; $k = 2$
$d_2 = 20 + 2(4) = 28$

5. $C_t = \$45 + \$2.25n$; $k = \$2.25$
$C_{29} = 45 + 2.25(29) = \$110.25$

6. $C_t = \$1000 + \$110n$; $k = \$110$
a) $C_{270} = 1000 + 110(270) = \$30\ 700$
b) $x = 300$ hectares

7. $C_t = \$200 + \$20n$; $k = \$20$
a) $C_{38} = 200 + 20(38) = \$960$
b) $n = 43$ hogs

C-1 The following are examples of inverse variation:
1, 2, 5, 6, 8 $(b = 1200/a)$, 9, 10

C-2,3 1. $k = 560$; $t_{100} = 5.6$h
2. $k = 300$; $d = 15$; $m = 10$
3. Since $V = k/R$ and $V = IR$ then $V = 6$
$k = 6R = 18$; $R = 1$
4. $k = 1200$; $m = \$8$
5. $k = 720$; $t = 20d$
6. $k = 225$; $m = 15$
7. $k = 1.27 \times 10^7$; $B = 3.763$ mm

D-1 1. Direct; 9 blows; $k = 3$
2. Inverse; $k = 2.7 \times 10^4$; $n = 216$
3. Partial; $k = 70¢$; $C_{150} = \$155.00$
4. Direct; \$1273.89
5. Inverse; $k = 2.56 \times 10^4$; $A = 1.024 \times 10^5$ or 102.4 k
6. Partial; $k = 56\ 1/4\ ¢$; $C_5 = \$3.81$

D-2 1. Partial
2. Direct; $y = 1/2\ x^2$
3. Direct
4. Inverse
5. Inverse
6. Partial

C.11. CONSUMER MATHEMATICS

A. Budgeting

2-b

1. Using the chart, "Breakdown of the Average Income Dollar," calculate the annual amounts that the following would spend in the different categories:

   a) a clerk-typist with an annual salary of $9800;
   b) an insurance representative with an annual salary of $21 800;
   c) a hair stylist with an annual salary of $18 200;
   d) an accountant with an annual salary of $38 500.

3

1. Using the chart, "Breakdown of the Average Income Dollar," calculate the monthly amounts that a family would spend given the annual income in each of the different categories:

   a) a surgeon with an annual salary of $72 500;
   b) a lawyer with an annual salary of $79 200;
   c) a mechanic with an annual salary of $21 800;
   d) a mail-order clerk with an annual salary of $10 100.

2. A family of three has a net income of $1424.80 a month. If they budgeted the income according to the following percentages, what is the amount allotted to each budget category?

Category	Percentage	Amount
Food	22%	
Household Operation	6%	
Clothing	5%	
Shelter	30%	
Transportation	15%	
Personal Needs	7.2%	
Savings	10%	
Miscellaneous	4.8%	

4-a

1. Dean Charles, an assistant elevator agent, receives a weekly salary of $595.00. After all deductions, his net weekly salary is $445.25. His wife receives a family allowance cheque for their three children which amounts to $26.93 per child. Their basic expenses are: mortgage payments of $525 per month; payments on a new car of $213.50; property taxes of $725 per year; food which averages $125 per week; life insurance on which he pays $60 every three months; utilities which average $80.00 per month; telephone which averages $10.40 per month. Prepare a Statement of Monthly Income and Expenditures for the Charles' family.

2. Glen and Sara Jones are both employed with monthly net incomes of $1375 and $825 respectively. They have purchased a townhouse with monthly mortgage payments of $485. Other expenses include property taxes of $1475 per year; food which averages $3600 per year; Glen has a life insurance policy with payments of $8.10 per week; utilities which include telephone average $172.50 per month. Prepare a Statement of Monthly Income and Expenditures for Glen and Sara.

3. Charlie and Bonny Wood are both employed. Charlie is the manager of the Mortgage Hill Bank with a salary of $680 per week. His net weekly salary after deductions is $491.82. Bonny is employed as a telephone operator with a net weekly salary of $281.42. They have four children. Bonny receives a monthly family allowance cheque of $107.72. Their basic expenses are: a first mortgage on their home with monthly payments of $531.50; a second mortgage with monthly payments of $201.65; payments on a new car of $237.75 per month; a weekly payment of $52.75 for a new boat; their property taxes amount of $1965.40 per year; food which averages $185 per week; life insurance policy of $90.60 every three months; homeowner's insurance policy of $248.70 per year; utilities (electricity, home heat, and water) which average $407.60 per month; telephone which averages $21.60 per month. Prepare a Statement of Monthly Income and Expenditures for Charlie and Bonny Wood.

4. Prepare a Statement of Yearly Income and Expenses for each of questions 1, 2 and 3.

4-c

1. With reference to question 4a-1, the Dean Charles' family; they have additional expenses which must be estimated for the year in order to maintain their standard of living. These expenses are: clothing, $725.00; entertainment, $250.00; Christmas, $230.00; newspapers and periodicals, $65.00; car maintenance, $630.00; gas and oil, $20 per week.

   a) Calculate the yearly amount left for further purchases and savings from the yearly estimated amount available for savings and other expenditures.
   b) Calculate the monthly amount available for additional purchases and savings.

2. Erika and Tom Elsimatesky have 2 children and live in rural Manitoba. Erika is employed with Camair and nets $201.60 per week. Her husband Tom is employed with No-Stud Const. Ltd. as a district manager and nets $410.50 per week. Erika receives the monthly family allowance cheques which amount to $26.93 per child. Their basic expenses include: monthly mortgage payments of $725.60; monthly automobile payments of $186.40; property taxes for the year of $1840.16; homeowner's insurance policy of $308.10 per year; utilities which average $311.60 per month; telephone which averages $11.30 per month; food which averages $135 per week.

   Additional expenses estimated for the year to maintain their standard of living are as follows: clothing $650; entertainment $180.00; vacation $470.00; car maintenance $560.00; special holidays (Christmas, etc.) $240.00; babysitter (full time) $49.00 per week; magazine subscriptions of $42.00 a year.

   a) Prepare a Statement of Monthly Income and Expenditures for the Elsimatesky Family.
   b) Prepare a Statement of Yearly Income and Expenses for the Elsimatesky Family.
   c) Calculate the yearly amount left for further purchases and savings from the yearly estimated amount available for savings and other expenditures.
   d) Calculate the monthly amount available for additional purchases and savings.

5 1. Calculate the reserve fund for questions 4a - 1, 2, 3; 4c - 2.

2. Prepare a Budget Balance Sheet for questions 4a - 1, 2, 3; 4c - 2, using the form provided.

Review Exercises

1. Elaine Soluk earns $220 a week "take home" pay. She shares an apartment with three other girls; they pay $560 a month rent; they spend $96 a week on food; their telephone bill is $8.80 a month; laundry for all four costs $5.60 a week. Elaine spends an average of $2.30 a day for lunch, five days a week; and transportation costs her $6.50 a week. If all apartment expenses are shared equally, prepare a statement of monthly income and expenses for Elaine.

2. Using the chart, "Breakdown of the Average Income Dollar," calculate the monthly and annual amounts that the following would spend in the different categories.

   a) Air traffic comptroller with an annual salary of $36 300;
   b) Carpenter with an annual salary of $23 400;
   c) Computer analyst with an annual salary of $48 200.

3. Dennis Murray earns $17 700 per year, which after deductions, provides him with a monthly cheque of $933.33. His wife earns a "take home" pay of $212.56 a week. The Murray's have recently bought a home on which they are making mortgage payments of $425.00 per month and monthly payments of $165.00 on a three-year bank loan which helped to finance the purchase. Taxes are $730.00 a year; heat is estimated at $740 a year; electricity and water bills are approximately $83.50 for a two-month period; the telephone bill averages approximately $10.50 a month; house insurance is $242.00 a year; and food costs are $105 a week. The Murrays have one child for whom Mrs. Murray receives the family allowance cheque. Family Allowance for the year amounts to $323.16. The Murrays are making payments of $212.00 a month on a one-year loan they made to finance the purchase of furniture. It is also necessary for Dennis Murray to drive his car to work and the basic expenses for this transportation is $45.00 a week; insurance for the car costs $288 per year.

   a) Complete a Statement of Monthly Income and Expenses for the Murrays.
   b) Complete a Statement of Yearly Income and Expenses for the Murrays.
   c) Estimated additional expenses per year are: clothing, $795.00; entertainment, $330; Christmas spending, $135; newspapers, books, and magazines, $125; further car expenses, $255; vacation (a planned trip to Barbados), $850. Estimate the yearly income available for this additional spending and the amount left over for any further purchases and savings?
   d) Calculate the monthly amount available for additional purchases and savings.
   e) Can the Murrays really afford the trip to Barbados?

4. Calculate the reserve fund for questions 1, 3.

5. Prepare a Budget Balance Sheet for question 3 using the form provided.

BUDGET FORM

STEP 1: INCOME

ANNUAL TAKE-HOME PAY _____ NET INCOME FOR THE MONTH _____

OTHER INCOME _____

ANNUAL NET INCOME _____

STEP 2: FIXED EXPENSES

	J	F	M	A	M	J	J	A	S	O	N	D
RENT												
UTILITIES												
TELEPHONE												
CAR DEBT												
CREDIT CARD DEBT												
OTHER DEBT												
CAR INS. PREMIUM												
LIFE INS. PREMIUM												
CAR MAINTENANCE												
MISCELLANEOUS												

TOTAL FOR THE YEAR _____

SET ASIDE FOR THE MONTH _____

STEP 3: LIVING ALLOWANCES FOR A MONTH

GROCERIES _____ ENTERTAINMENT _____

LAUNDRY _____ PERSONAL _____

CLOTHING _____ OTHER _____

GASOLINE _____

TOTAL LIVING ALLOWANCE FOR A MONTH _____

STEP 4: RESERVE FUND

RESERVE FUND (TWO MONTHS' PAY) WILL GROW UNTIL IT REACHES _____

RESERVE FUND PAYMENTS FOR THE MONTH _____

STEP 5: BUDGET BALANCE SHEET

NET INCOME FOR A MONTH (STEP 1) _____

"SET ASIDE" FOR A MONTH (STEP 2) _____

LIVING ALLOWANCE FOR A MONTH (STEP 3) _____

RESERVE FUND PAYMENTS FOR A MONTH (STEP 4) _____

SAVINGS FOR A MONTH (TOTAL 2, 3, 4, AND DEDUCT FROM 1) _____

BUDGET FORM

STEP 1: INCOME

ANNUAL TAKE-HOME PAY _____ NET INCOME FOR THE MONTH _____

OTHER INCOME _____

ANNUAL NET INCOME _____

STEP 2: FIXED EXPENSES

	J	F	M	A	M	J	J	A	S	O	N	D
RENT												
UTILITIES												
TELEPHONE												
CAR DEBT												
CREDIT CARD DEBT												
OTHER DEBT												
CAR INS. PREMIUM												
LIFE INS. PREMIUM												
CAR MAINTENANCE												
MISCELLANEOUS												

TOTAL FOR THE YEAR _____

"SET ASIDE" FOR THE MONTH _____

STEP 3: LIVING ALLOWANCES FOR A MONTH

GROCERIES _____ ENTERTAINMENT _____

LAUNDRY _____ PERSONAL _____

CLOTHING _____ OTHER _____

GASOLINE _____

TOTAL LIVING ALLOWANCE FOR A MONTH _____

STEP 4: RESERVE FUND

RESERVE FUND (TWO MONTHS' PAY) WILL GROW UNTIL IT REACHES _____

RESERVE FUND PAYMENTS FOR THE MONTH _____

STEP 5: BUDGET BALANCE SHEET

NET INCOME FOR A MONTH (STEP 1) _____

"SET ASIDE" FOR A MONTH (STEP 2) _____

LIVING ALLOWANCE FOR A MONTH (STEP 3) _____

RESERVE FUND PAYMENTS FOR A MONTH (STEP 4) _____

SAVINGS FOR A MONTH (TOTAL 2, 3, 4, AND DEDUCT FROM 1) _____

B Taxation

Exercises in taxation are provided in the Teacher's Manual - Teaching Taxes. Teachers should be aware that Revenue Canada provides these booklets nation wide. To accommodate all provinces the tax tables provided in the Workbook do not correspond exactly to the actual Manitoba situation and, therefore, they should not be used when filing an income tax return with Revenue Canada, Taxation.

Included in this package is one additional question that could be attempted by the student.

I 1. Tommy Kramer and his wife Beth live in an apartment in Winnipeg. The Kramers have two children, Glen and Carolyn, ages 7 and 4 respectively. The Kramers rent a two bedroom apartment for $390 a month. Mrs. Kramer worked part time during the year and earned $1350. Mr. Kramer contributed $900 into a Registered Retirement Savings Plan during the year. Tommy also paid $240 for union dues during the year. In addition to the previous information, Mr. Kramer will use the following T4, TFAI and T5 slips that are included to complete his return. Calculate the refund or balance due for the family. (NOTE: Mrs. Kramer will be required to complete a Schedule 10 - Child Tax Credit - in order to calculate the total family refund or balance due.)

II-I 1. Calculate the rate of assessment in cents on a dollar, to the nearest tenth of a cent, for each of the following years in the Town of Limpet:

	Year	Assessed Value of Real Property	Budget Requirement	Rate of Assessment
a)	1980	$1 900 000	$62 000	
b)	1981	$1 940 000	$70 000	
c)	1982	$2 030 000	$76 000	

2. Calculate the rate of assessment in the form of a percent rate, to the nearest tenth of a percent, for each of the years indicated in the City of Bigg:

	Year	Assessed Value of Real Property	Budget Requirement	Rate of Assessment
a)	1978	$890 000 000	$78 000 000	
b)	1980	$942 000 000	$86 000 000	
c)	1982	$965 000 000	$102 000 000	

Revenue Canada Taxation | Revenu Canada Impôt

T5

Supplementary – *Supplémentaire*

Rev. 81

STATEMENT OF INVESTMENT INCOME
ÉTAT DES REVENUS DE PLACEMENT

Year / Année	Dividends from Taxable Canadian Corporations — Dividendes de sociétés canadiennes imposables: (A) Actual Amount of Eligible Dividends — Montant réel des dividendes admissibles	(B) Taxable Amount of Eligible Dividends — Montant imposable des dividendes admissibles	(C) Federal Dividend Tax Credit — Dégrèvement fédéral pour dividendes	(D) Eligible Interest from Canadian Sources — Intérêts admissibles de sources canadiennes	(I) Capital Gains Dividends — Dividendes sur gains en capital	(J) Gross Foreign Income — Revenus étrangers bruts
1982				$146.33		
	(E) Actual Amount of Ineligible Dividends — Montant réel des dividendes non admissibles	(F) Taxable Amount of Ineligible Dividends — Montant imposable des dividendes non admissibles	(G) Royalties from Canadian Sources — Redevances de sources canadiennes	(H) Other Income from Canadian Sources — Autres revenus de sources canadiennes	(K) Foreign Tax Paid — Impôt étranger payé	

RECIPIENT SURNAME FIRST AND FULL ADDRESS
BÉNÉFICIAIRE NOM DE FAMILLE D'ABORD ET ADRESSE COMPLÈTE

→ KRAMER, TOMMY
567 WILLOW PARK DRIVE
WINNIPEG, MB
R3J, 0X3

Social Insurance Number
Numéro d'assurance sociale
591 | 034 | 703

NAME AND ADDRESS OF PAYER (Must appear on each slip)
NOM ET ADRESSE DU PAYEUR (À inscrire sur chaque feuillet)

IMPERIAL BANK of COMMERCE
WINNIPEG, MB.

• For Recipient
Attach to your Income Tax Return
SEE INFORMATION ON REVERSE

• *Pour le bénéficiaire – Annexer à votre déclaration d'impôt sur le revenu*
VOIR LES RENSEIGNEMENTS AU VERSO

2

Revenue Canada Taxation | Revenu Canada Impôt

TFA1
Supplementary
Supplémentaire
Rev. 81

STATEMENT OF FAMILY ALLOWANCES
RELEVÉ DES ALLOCATIONS FAMILIALES

10. YEAR	11. TOTAL ALLOWANCES / TOTAL DES ALLOCATIONS	12. NO OF CHILDREN / NOMBRE D'ENFANTS	13. DATE OF BIRTH OF EACH CHILD / DATE DE NAISSANCE DE CHAQUE ENFANT									
			MO.	YEAR	MO.	YEAR	MO.	YEAR	MO.	YEAR	MO.	YEAR	MO.	YEAR
198-	$ C46 12	2	6	75										
			12	77										
10. ANNÉE	11. TOTAL DES ALLOCATIONS	12. NOMBRE D'ENFANTS	MOIS	ANNÉE	MOIS	ANNÉE	MOIS	ANNÉE	MOIS	ANNÉE	MOIS	ANNÉE	MOIS	ANNÉE

RECIPIENT SURNAME FIRST AND FULL ADDRESS
BÉNÉFICIAIRE NOM DE FAMILLE D'ABORD ET ADRESSE COMPLÈTE

KRAMER, BETH
507 WILLOW PARK DRIVE
WINNIPEG, MAN.
R3J, 0x3.

14A. RECIPIENT'S SOC. INS. NO	14B. FAMILY ALLOWANCE NO.
513 : 2 : 515	3 0 3 4 7
14A. N° D'ASS. SOC. DU BÉNÉFIC.	14B. N° D'ALLOCATION FAMILIALE

14C. OTHER PARENT'S NAME AND SOCIAL INSURANCE NUMBER

Terry Kramer	587 : 634 : 103
14C. NOM ET N° D'ASSURANCE SOCIALE DU CONJOINT	

ISSUED BY HEALTH AND WELFARE CANADA
DÉLIVRÉ PAR SANTÉ ET BIEN-ÊTRE SOCIAL CANADA

SEE INFORMATION ON REVERSE FOR REPORTING REQUIREMENT
VOIR LES RENSEIGNEMENTS AU VERSO POUR DÉCLARATION

2

Revenue Canada Taxation | Revenu Canada Impôt

T4 - 1981
Supplementary - Supplémentaire

STATEMENT OF REMUNERATION PAID
ÉTAT DE LA RÉMUNÉRATION PAYÉE

(C) TOTAL EARNINGS BEFORE DEDUCTIONS / GAINS TOTAUX AVANT DÉDUCTIONS	(D) EMPLOYEE'S PENSION CONTRIBUTION CANADA PLAN / COTISATION DE PENSION (EMPLOYÉ) DU CANADA	QUEBEC PLAN / DU QUÉBEC	(E) U.I. PREMIUM / PRIME D'A.-C.	(F) REGISTERED PENSION PLAN CONTRIBUTIONS / COTISATIONS AU RÉGIME DE PENSIONS	(G) INCOME TAX DEDUCTED / IMPÔT SUR LE REVENU DÉDUIT	(H) U.I. INSURABLE EARNINGS / GAINS ASSURABLES A.-C.	(I) C.P.P. CONTRIBUTORY EARNINGS / GAINS COTISABLES POUR R.P.C.	(J) EXEMPT C.P.P. R.P.C. / EXEMPT U.I. A.-C. / EXEMPTION
$ 33,246 00	$ 334 40		$ 244 37	$ 1500 00	$ 4,301 50	$ 15,200 00	$ 12,500 00	

BOX (C) AMOUNT INCLUDES ANY AMOUNTS IN BOXES (H), (I), (K) AND (L) / LE MONTANT DE LA CASE (C) COMPREND TOUS MONTANTS FIGURANT AUX CASES (H), (I), (K) ET (L)	(K) TAXABLE ALLOWANCES AND BENEFITS / ALLOCATIONS ET PRESTATIONS IMPOSABLES	(L) EMPLOYMENT COMMISSIONS / COMMISSIONS D'EMPLOI	(M) PENSION PLAN REGISTRATION NUMBER / N° D'ENREGISTREMENT DU RÉGIME DE PENSIONS
	0	0	11362

* If different from Box (C) - S'ils sont différents de la Case (C)

EMPLOYEE - EMPLOYÉ
SURNAME FIRST AND GIVEN NAMES, USUAL FIRST NAME AND INITIALS AND FULL ADDRESS
NOM DE FAMILLE D'ABORD LES PRÉNOMS, PRÉNOM USUEL ET INITIALES ET ADRESSE COMPLÈTE

KRAMER, TAMMY
567 WILLOWPARK DRIVE
WINNIPEG, MAN.
R2J, 0X3

(A) PROVINCE OF EMPLOYMENT / PROVINCE D'EMPLOI	(B) SOCIAL INSURANCE NUMBER / N° D'ASSURANCE SOCIALE	(N) EMPLOYEE NO. / N° DE L'EMPLOYÉ
MAN	554 634 153	

NAME AND ADDRESS OF EMPLOYER - NOM ET ADRESSE DE L'EMPLOYEUR

VIKING SPORTSCRAFT
BOX 352
WINNIPEG, MB

• Attach to your 1981 Income Tax Return
SEE INFORMATION ON REVERSE

• Annexer à votre déclaration d'impôt sur le revenu de 1981
VOIR LES RENSEIGNEMENTS AU VERSO

2

3. Calculate the rate of assessment in terms of mills, to the nearest whole mill for each of the years listed, in the rural municipality of St. Ives.

	Year	Assessed Value of Real Property	Budget Requirement	Rate of Assessment
a)	1970			
b)	1975			
c)	1980			

II-2 1. Complete the following table:

	Property	Assessed Value	Tax Rate	General Tax
a)	Building Lot	$ 7 200	4¢	_____
b)	House	$ 44 800	2.5¢	_____
c)	Farm	$165 000	2¢	_____
d)	Factory	$180 000	5¢	_____
e)	Duplex	$ 67 500	3¢	_____

2. The Watsons have a new house for which they paid $62 000. It is assessed at $25 000. If the tax rate in their area is 4¢ on the dollar of assessed value, find the amount of the Watson's general tax for the year.

3. Alan Oxford receives his tax bill which states that the assessed value of his property amounts to $14 000, and that taxes are calculated on the basis of 7¢ on a dollar. There is a local improvement tax for ornamental street lighting, which amounts to $93.50 for the year. Calculate Mr. Oxford's total tax bill for the year.

4. Complete the following table:

	Property	Assessed Value	Tax Rate	General Tax
a)	Roominghouse	$ 46 000	5%	_____
b)	Office building	$680 000	3%	_____
c)	House	$ 24 000	4%	_____
d)	Lot	$ 5 000	6%	_____

5. The Martins own a house valued at $52 000. If it is assessed at 40% of market value, and the tax is assessed at 5% of assessed value, calculate the general tax bill for the year.

6. The Renchuks own an office building with a market value of $220 000. The property is assessed at 30% and the tax rate is 5%. In addition there are two local improvements to be paid: paving, which costs $1500, spread over 10 years, and ornamental street lighting which costs $860, also distributed over a 10 year period. Calculate the Renchuk's total tax bill for this year.

7. Complete the following table:

Property	Assessed Value	Tax Rate	General Tax
a) Bungalow	$16 000	62 mills	_____
b) Factory	$58 000	64 mills	_____
c) Commercial Lot	$ 5 000	38 mills	_____
d) Duplex	$27 500	60 mills	_____

8. Complete the following table

Property	Market Value	Assessment Value	Assessed Value	Mill Rate	General Tax	Local Improvements	Total Tax Bill
a) Cottage	$28 000	40%	_____	52	_____	0	_____
b) Bungalow	$52 000	42%	_____	55	_____	135.20	_____
c) Duplex	$88 000	48%	_____	60	_____	168.85	_____
d) Factory	$25 600	50%	_____	62	_____	345.50	_____

9. Janet Guthrie purchased a new house for $38 000. It was assessed for tax purposes at 40%, and the mill rate was set for the year at 61. If there is also a local improvements tax of $128.50 for sidewalks, calculate the total tax for the year.

10. The Smiths purchased a $50 000 house. In their first year of occupancy, the assessment was at a rate of 30%, and the mill rate for that year was 68. The property was re-assessed the next year at a new rate of 35%, and the mill rate increased to 70 mills. Find the amount of the general tax increase from year one to year two.

III-1 Hidden Taxes

1. Calculate the federal sales tax on the following goods:

Item	Number	Unit Cost	Federal Sales Tax
a) Refrigerators	10	$310	_____
b) Television	8	$430	_____
c) Bicycle	100	$ 83	_____
d) Sofas	20	$600	_____
e) Bookcases	5	$250	_____

2. What is the federal sales on a shipment to the retailer of 25 chairs at a unit cost of $32.50?

III-2 Below is a table of custom tariffs. Use the table to answer the questions that follow.

Customs Tariffs

Goods	British Pref.	Most Favoured Nation	General	Goods	British Pref.	Most Favoured Nation	General
Automobiles	Free	15%	27.5%	Jewellery, gold	20 %	25 %	45 %
Baskets, wire	10%	17.5 %	35 %	Jewellery, not gold	20 %	25 %	45 %
Bottles, glass	15 %	15 %	32.5 %	Pencils, lead	10 %	15 %	35 %
Brushes, artist	Free	17.5 %	30 %	Perfumery (no-alcohol)	15 %	15 %	40 %
Cabinets, wood	15 %	20 %	45 %	Phonographs and parts thereof	10 %	15 %	25 %
Combs	10 %	20 %	30 %	Radio and television apparatus	Free	15 %	25 %
Cellophane, not printed	10 %	10 %	20 %	Tobacco, cut per pound	40¢	40¢	75¢
Drills, hand	10 %	15 %	35 %	Watches	15%	20%	35%
Fabrics, wholly of cotton	22.5 %	22.5 %	35 %				

NOTE: To calculate the total cost of imports:
- A. Find the total invoice in Canadian funds.
- B. Calculate the customs tariffs that apply.
- C. Add (A) and (B) to find the duty-paid value.
- D. Find the federal sales tax on the duty-paid value.
- E. Find the excise tax, if required, on the duty-paid value.
- F. Add (C), (D) and (E) to find the total cost of the import.

1. Wood cabinets costing $250 each were imported from Britain. The retailer purchased ten of these cabinets. What would be the custom tariff added on to the price of these cabinets?

2. A local retail store decided to import 100 bottles of perfume at a price of $5.00. The country from which the perfume was purchased was in the "Most Favoured Nation" category. After this tax was added to the cost an additional 15% federal sales tax was added. What was the total cost of the shipment of perfume?

3. A local hardware store must decide between two hand drills. One is available from Britain at a price of $17 plus custom tariff plus federal sales tax. A second drill is available from the United States at $15.50 plus custom tariff plus federal sales tax. Which of the two prices would have a lower cost price to the merchant importing the goods?

4. Phonograph records are imported from the United States at a price of $3.00. A record store purchased 1000 records. The records were subject to custom tariffs plus federal sales tax. What would be the total cost of the records to the record store?

III-3 Below is a table of Excise Taxes. Use the table to answer the questions that follow.

Excise Taxes

Goods	Rate
Lighters	10¢ per lighter
Coin, disc or token operated games or amusement devices	10%
Smoker's accessories	10%
Cigars	20.5%
Matches	10%
Clocks and watches	10% ← of the amount by which the sales price or duty-paid value of each watch exceeds $50.00.
Jewellery	10%
Cigarettes (for each five cigarettes or fraction of five)	3¢
Tobacco, manufactured (per pound)	90¢
Playing cards (per pack)	20¢
Wine (per gallon) less than 7% alcohol	25¢
Champagne and sparkling wines (per gallon)	$2.50

1. An amusement store purchased 100 decks of playing cards at 50 cents per pack. The store paid the required excise tax plus the federal sales tax. What would be the total cost of the playing cards to the store?

2. A cigarette company in Canada imported 1500 pounds of tobacco from the United States at $2.50 a pound. The company had to pay the necessary custom tariff, plus the excise tax, plus the required federal sales tax. Calculate the total cost to import the tobacco.

3. A jewellery store imported a shipment of watches from Japan, a most favoured nation. The watches cost $80 Canadian. The watches were subject to custom tariffs, excise duties and federal sales tax. Calculate the total cost to the jewellery store if they imported 25 of these watches.

4. A jewellery store imported a shipment of $1500 worth of gold jewellery from Afghanistan, which falls into the general custom tariff category. Calculate the total cost to the store if the jewellery was subject to custom tariffs, federal sales tax, and excise taxes.

C Insurance

Note: All questions pertain to tables in section on insurance in the guide.

C-2b 1. Using the premium rate table, calculate the annual premium for each of the following:

	Age At Time Of Issue	Type of Policy	Amount of Policy	Annual Premium
a)	15	10 year term	$ 4 000	_____
b)	40	whole term	$10 000	_____
c)	25	15 year term	$15 500	_____
d)	60	20 year endowment	$50 000	_____
e)	35	30 year limited payment	$27 550	_____
f)	50	15 year term	$33 500	_____
g)	20	20 year limited payment	$20 000	_____
h)	55	10 year term	$100 000	_____
i)	30	whole life	$250 000	_____
j)	45	30 year limited payment	$300 500	_____

2. Find the periodic payment on each policy.

	Age At Time Of Issue	Type of Policy	Amount of Policy	Period	Periodic Payment
a)	25	whole life	$ 4 500	semiannually	_____
b)	40	10 year term	$ 25 500	quarterly	_____
c)	15	20 year limited payment	$ 45 000	monthly	_____
d)	55	20 year endowment	$ 60 000	semiannually	_____
e)	20	30 year limited payment	$100 000	monthly	_____
f)	30	15 year term	$ 75 550	quarterly	_____

3. Shaylin Kristin purchased a $10 000 10 year term policy at age 20. If she had purchased the policy at age 15, how much less would her annual premiums have been.

4. At age 40, how much 10-year term insurance could be purchased for an annual premium of $113.05?

5. If the annual premium of a whole life policy is $27.60 per $1000 of insurance, how much insurance can Jason buy for no more than $552 per year.

6. Tim took out a 20 year Limited payment insurance policy at age 35 for $105 000. He died after paying premiums for 18 years. What is the total of the premiums he had paid into the insurance plan? How much would his beneficiary receive?

C-3c 1. Refer to the Select Table - Building to compute the annual premium of each fire insurance policy.

Policy	Amount of Insurance	Location	Deductible	Annual Premium
a)	$ 40 000	Metro Wpg.	$ 50	_____
b)	$ 65 000	Table 1	$100	_____
c)	$ 80 000	Table 3	$ 50	_____
d)	$ 95 000	Metro Wpg.	$100	_____
e)	$150 000	Table 2	$ 50	_____
f)	$250 000	Table 3	$ 50	_____
g)	$ 30 000	Table 1	$100	_____
h)	$325 000	Metro Wpg.	$100	_____

2. Find the difference in annual premiums.
(Note: Premiums are $50 deductible.)

	Amount of Insurance	Location: Metro Wpg.	Location: Table 3	Difference
a)	$ 40 000	$_____	$_____	$_____
b)	$140 000	$_____	$_____	$_____
c)	$ 85 000	$_____	$_____	$_____
d)	$ 65 000	$_____	$_____	$_____

3. Darrin lives in Metro Wpg. and carries a $75 000 fire insurance policy on his house. Danny lives in a town classified as Table 2 with a $60 000 fire insurance policy. Each has $100 deductible. What is the difference in premiums?

C-3d

1. From the Homeowner "Standard Rates" table, calculate the annual premium for each of the following homeowner's policies.

	Basic Coverage	Location	Premium
a)	$ 95 000	Table 1 $50 deductible	_____
b)	$150 000	Table 3 $50 deductible	_____
c)	$125 000	Table 2 $100 deductible	_____
d)	$ 55 000	Metro Wpg. $100 deductible	_____

2. Sherril Charles purchased a $85 000 homeowner's Standard preferred policy and a $35 000 contents 1007 policy. She lives in town grade 2. Using the Manitoba Homeowner's table and tenants package, calculate her annual premium?

3. Using the homeowner's policy based as a percent of the amount of insurance, calculate the amount of coverage a person would have for each type of risk on $140 000 coverage.

4. Calculate how much the insured would collect in the following circumstances using the table on a percent of the amount of insurance:

   a) Basic coverage $45 000; $24 000 damage to house, owner forced to live in a motel which cost $789.00.

   b) Basic coverage $90 000. Garage damaged to extent of $10 000; fence damaged $75.00; $40 000 damage to house.

c) Basic coverage $41 000. Kitchen fire caused $8000 damage to house. Smoke and water damage to contents estimated at $4300.

d) Basic coverage $60 000. Home completely destroyed by fire. Damage evaluated as follows: dwelling $69 000; contents $25 500; additional living expenses $900.00.

C-3e 1. Calculate the loss paid by the insurance company when the following losses occur, assuming that all policies have an 80% co-insurance clause.

	Amount of Insurance	Property Value	Amount of Loss	
a)	$60 000	$75 000	$45 000	_____
b)	29 000	35 000	5 000	_____
c)	25 000	38 000	19 500	_____
d)	40 000	48 000	42 000	_____
e)	20 000	34 000	12 500	_____
f)	38 000	59 000	30 000	_____
g)	25 000	46 000	22 000	_____
h)	16 000	26 000	5 000	_____

2. David Stiller's office building is valued at $46 000 and is insured against fire for $43 000. If the fire destroys one room to the extent of $22 000, how much of the loss must the insurance company pay?

C-4 Automobile

Note: Obtain various rate schedules through MPIC to compare rates of different areas and vehicle makes.

Unit Review

For solving many of the problems, refer to the tables in this unit.

1. Calculate the annual premium for a person at the age of 31 who wanted to take out a 15 year term insurance for $95 000.

2. Find the annual premium on a $50 000 whole life policy issued at age 21 for premiums payable semiannually, quarterly, and monthly.

3. A straight life insurance policy costs $11.35 per thousand per year.

   a) How much would the annual premium be for a $15 500 policy?

   b) How much would have been paid to the insurance company after 11 years?

4. How much more will the premiums be on a $10 500, 20 year limited payment life policy issued at age 35 than on a 30 year limited payment life policy?

5. Lillian purchased a homeowner's policy with basic coverage of $87 350 for her home. Based on a percent of insurance on the dwelling, calculate the total coverage she has including outbuildings, trees, contents, theft, and living expenses.

6. How much will Casey save each year on his $20 000, 20 year endowment policy by making annual payments rather than quarterly payments? The policy was issued at age 40.

7. How much will an insurance company pay for a loss of $26 000 on a building valued at $30 000, if it is insured for $24 000 with an 80% co-insurance clause?

D. Mortgages

NOTE: The section on property taxes should be covered before starting mortgages, due to calculations of taxes on mortgaged property. The following project may be attempted to introduce this module.

1

1. Prices of homes have increased steadily and it is often stated that home ownership is one of the better investments in our economy. Make a comparison for each of the past five years of the average costs of the following by contacting your local real estate agent (or board).

   a) a new home
   b) an old home resale
   c) monthly rent for an apartment
   d) monthly rent for a townhouse
   e) monthly rent for a condominium

2. a) What has been the average increase in the price of a new home in the past five years?
   b) What has been the percentage increase in the price of a new home in the past five years?

3. a) What has been the average increase in the price of an old home resale in the past five years?
   b) What has been the percentage increase in the price of an old home resale in the past five years?

2

1. Casey and Evelyn bought a house from Mr. Jones for $85 500. They had only $15 000 for a down payment and had to take over the existing mortgage of $49 700 at 12 1/2 % held by the Bank of Montreal. They acquired the remainder of the purchase price by taking out another mortgage with Mr. Charles at 21%.

   a) Define a mortgage.
   b) Who is the mortgagor?
   c) Who is the first mortgagee?
   d) Who is the second mortgagee?
   e) What is the amount of the second mortgage?
   f) What is the interest rate on the first mortgage?
   g) What is the interest rate on the second mortgage?
   h) Why is one mortgage rate higher than the other?

3a NOTE: As interest rates are constantly changing, the current rates may be substituted in these exercises.

1. Alvin bought a home for $32 000. He paid $26 000 down and gave the seller a $6000, 8 year mortgage at 18%. The mortgage was dated March 1. Alvin was to make regular yearly payments of $750 plus interest on the unpaid balance beginning on July 31. Prepare a schedule of mortgage payments for the eight years.

Schedule of Mortgage Payments

Date	Regular Payment			Unpaid Balance	Owner's Equity
	Principal	Interest	Total		

2. Prepare an amortization schedule for the first payments from the following information:

   Date of mortgage, March 1
   Amount of Mortgage on March 1, $26 000
   Monthly payments on the principal, $250 beginning March 31
   Interest on the mortgage, 19 3/4 %

   a) How much interest is paid in the first 6 months?
   b) How much principal is paid in the first 6 months?
   c) How long will it take to pay off the mortgage?

Schedule of Mortgage Payments

Date	Regular Payment			Unpaid Balance	Owner's Equity
	Principal	Interest	Total		

3. Lorin bought a $65 000 property for $15 000 down and signed a mortgage agreeing to pay the remainder of the purchase price by equal quarterly payments of $500 on principal plus interest on the unpaid balance at 22 1/2 %.

   a) Prepare a schedule to show Lorin's payments for the first two years.
   b) How much will Lorin still owe at the end of 10 years?
   c) How long will it take Lorin to completely pay the debt?

Schedule of Mortgage Payments

Date	Regular Payment			Unpaid Balance	Owner's Equity
	Principal	Interest	Total		

4. Prepare an amortization schedule for the first 6 payments using the following information:

   Date of mortgage, August 15
   Amount of mortgage on August 15, $25 000
   Monthly payments on the principal, $320 beginning September 15
   Interest on the mortgage, 18 3/4 % per annum
   Down payment was $20 000

   a) Amortization schedule
   b) How much interest was paid in the first 6 months?
   c) How much principal was paid in the first 6 months?
   d) How long will it take to pay off the mortgage?
   e) What will the owner's equity be at the end of 5 years?

Schedule of Mortgage Payments

Date	Regular Payment			Unpaid Balance	Owner's Equity
	Principal	Interest	Total		

3b-i 1. A home was purchased for $65 000. The buyer made a down payment of $27 000 with the balance of the purchase price to be mortgaged over 30 years. Regular monthly payments of $685.00 are to be made beginning January 15. The interest rate is 19 3/4 % per annum. Prepare an amortization schedule for the first 6 payments.

a) How much interest was paid in the first 6 months?
b) How much principal was paid in the first 6 months?

Schedule of Mortgage Payments

Date	Regular Payment			Unpaid Balance	Owner's Equity
	Principal	Interest	Total		

3b-ii 1. Using an amortization table, find the equal monthly payment required to amortize the following mortgages.

	Amount of Mortgage	Rate of Interest	Amortization Period	
a)	$ 6 000	12%	7 years	_____
b)	42 200	20 1/4 %	10 years	_____
c)	64 500	23 1/2 %	20 years	_____
d)	22 000	18 1/2 %	15 years	_____
e)	95 500	19%	18 years	_____
f)	120 500	11 1/4%	30 years	_____
g)	73 400	23 3/4 %	17 years	_____
h)	80 000	24%	25 years	_____

2. A $72 000 home was purchased July 1. The buyer made a $30 000 down payment and gave a 10 year mortgage for the balance of the purchase price, which was to be paid in equal monthly payments beginning August 1. The interest rate was 18 1/2 % per annum.

   a) Find the equal monthly payment.
   b) Prepare an amortization schedule for the first 7 payments.

Schedule of Mortgage Payments

Date	Regular Payment			Unpaid Balance	Owner's Equity
	Principal	Interest	Total		

3. Vic and Agnes had a $15 000, 15 year mortgage at 19 1/2 % payable in equal monthly installments.

   a) What was their monthly payment?
   b) What was the interest on the first payment?

4. Property was purchased for $42 000 in which a $25 000 down payment was made and the remainder was to be paid back in equal monthly payments including principal and interest at 20 1/2 % over 10 years.

   a) What was the monthly payment?
   b) What was the interest on the first payment?
   c) How much will be paid in 10 years?
   d) What will be the total amount of interest paid after the last payment is made?

5. Ed purchased a piece of property for $42 000. Under the sales terms, $12 000 was required as a down payment with the remainder to be paid under a 25 year mortgage at 12% per annum.

   a) Calculate the equal monthly payment required to pay off the mortgage?
   b) How much would Ed pay for the property over the life of the mortgage?

6. Audrey purchases a home valued at $83 000. She is required to make a $20 000 down payment with the balance to be paid off on mortgages. The first mortgage, which her father agrees to hold, is valued at $55 000, bears interest at the rate of 12% per annum and is to be repaid monthly over a period of 30 years. The second mortgage for the balance is a 18 3/4 % mortgage to be paid over 10 years.

   a) What is her monthly payment for the first mortgage?
   b) What is her monthly payment to cover the second mortgage?
   c) How much will she pay for the home over the 30-year period?

4 Prepare a statement of adjustments for each of the following:

1. a) Closing date is April 15.
   b) Purchase price of property is $31 000.
   c) The buyer made a $1000 deposit at the time the offer to purchase was signed.
   d) The buyer is assuming the existing mortgage at 18%. The unpaid balance on April 15 is $16 000.
   e) Taxes are $1540 per year which were paid in full.
   f) A 3 year premium for fire insurance was paid on October 15 of the previous year. The premium was $3.80
   g) The last quarterly water bill of $40.80 was paid on February 28.

Particulars	Charged to Buyer	Credited to Buyer

2. a) Purchase price of property is $36 800.
   b) Closing date is July 31.
   c) The obligation of paying off an existing mortgage on the property is assumed by the purchaser. The balance owing on the mortgage at the end of the preceding month was $22 750. Equal monthly payments are made at the end of each month including interest at 19 1/2 % per annum.
   d) No taxes have been paid for the current year. The property is assessed at $12 400 and the tax rate is 62 mills.
   e) A fire insurance policy, which has been in effect 6 months as of the closing date, is to be transferred to the purchaser. The premium on the policy was $320 for 1 year and was prepaid.
   f) The water rates are paid quarterly, and were paid in full up to September 30. The bill was $36.20.
   g) A deposit of $1500 was made by the purchaser.

Particulars	Charged to Buyer	Credited to Buyer

3. a) Closing date is November 30.
   b) Purchase price is $86 000.
   c) Deposit on purchase price is $5000.
   d) Taxes for the year have been paid in full. Assessment is $34 000 and the rate is 52 mills.
   e) The purchaser assumes a mortgage of $62 000 at 18%. The last payment was made on October 30.
   f) The house is heated by an oil furnace. The full tank contains 188 L on the closing date. The purchaser agrees to pay for this oil at 31.2¢/L.
   g) A fire insurance policy of $72 000 is transferred to the purchaser. The yearly premium was $490 which was paid 6 months prior to the closing date.
   h) Quarterly water rates have been paid up to September 30. The bill at that time was $53.40.
   i) The seller agrees to accept from the purchaser a second mortgage on the property for $18 000 at 22 1/2 % interest on the closing date.

Particulars	Charged to Buyer	Credited to Buyer

Review Exercises

1. Walter bought a $48 000 home for $35 000 down and gave the seller a $13 000 mortgage at 10%. The mortgage was dated May 1. Walter was to make regular half-yearly payments of $1000 plus interest on the unpaid balance beginning June 1. Complete an amortization schedule for Walter's first four payments.

2. Earl bought a $48 000 property for $19 000 down and signed a mortgage agreement to pay the remainder of the purchase price by equal quarterly payments on principal of $450 plus interest on the unpaid balance at 11 1/4 %.

   a) Complete a schedule showing Earl's payments for the first year.
   b) How much will Earl still owe on his mortgage at the end of 5 years?
   c) How long will it take Earl to pay off the mortgage at the present rate?

Schedule of Mortgage Payments

Date	Regular Payment			Unpaid Balance	Owner's Equity
	Principal	Interest	Total		

3. A $45 000 home was purchased on September 1. The buyer made a $20 000 down payment and gave a 20 year mortgage for the balance of the purchase price, which was to be paid in equal monthly payments, beginning October 1. The interest rate was 11% per annum.

   a) Using an amortization table, find the equal monthly payment.
   b) Prepare an amortization schedule for the first four payments.

Schedule of Mortgage Payments

Date	Regular Payment			Unpaid Balance	Owner's Equity
	Principal	Interest	Total		

4. A mortgage of $15 000 is to be paid off with interest over a period of 30 years. The monthly payment is $118.95 including principal and interest. What is the interest rate on the mortgage?

5. Prepare a statement of adjustments using the following information:
   a) Closing date, March 15.
   b) Purchase price, $53 000.
   c) The purchaser assumes the existing mortgage. The unpaid balance on February 28 was $32 850. Equal monthly payments including principal and interest at 10% are to be made at the end of each month.
   d) Deposit on purchase price, $1000.
   e) No taxes have been paid. The property was assessed at $15 000, and the tax rate is 68.3 mills.
   f) The $41 000 fire insurance policy is to be transferred to the new owner. The $156.00 premium for the 3-year policy was paid on September 15 of the previous year.
   g) The last quarterly water bill of $28.75 was paid 2 months ago.

Particulars	Charged to Buyer	Credited to Buyer

6. Prepare a statement of adjustment using the following information:
   a) Closing date, May 1.
   b) Purchase price, $43 600.
   c) Deposit, $950.
   d) The purchaser will assume the existing 10 1/2 %, $30 675 mortgage. The last monthly payment was made April 15.
   e) Taxes were based on an assessment of $18 000 and a tax rate of 58.2 mills. Taxes were paid to September 1.
   f) The last premium on the $30 000 fire insurance policy was paid on December 1 of the previous year. The insurance cost was $1.55 per $100 of coverage.
   g) The full tank contained 300 L of oil worth 29.7¢/L.

Particulars	Charged to Buyer	Credited to Buyer

Answer Key

A. Budgeting

2-b 1. a)

Shelter	$2450
Food	2940
Clothing	980
Transportation	1176
Health & Personal Care	784
Recreation	588
Insurance	490
Savings	392
Total	$9800

b)

Shelter	$ 7 950
Food	9 540
Clothing	3 180
Transportation	3 816
Health & Personal Care	2 544
Recreation	1 908
Insurance	1 590
Savings	1 272
Total	$31 800

c)

Shelter	$ 4 550
Food	5 460
Clothing	1 820
Transportation	2 184
Health & Personal Care	1 456
Recreation	1 092
Insurance	910
Savings	728
Total	$18 200

d)

Shelter	$ 9 625
Food	11 550
Clothing	3 850
Transportation	4 620
Health & Personal Care	3 080
Recreation	2 310
Insurance	1 925
Savings	1 540
Total	$38 500

3 1.

		Annual	Monthly
a)	Shelter	$18 125	$1 510.42
	Food	21 750	1 812.50
	Clothing	7 250	604.17
	Transportation	8 700	725.00
	Health & Personal Care	5 800	483.33
	Recreation	4 350	362.50
	Insurance	3 625	302.08
	Savings	2 900	241.67
	Total	$72 500	$6 041.67

Annual Salary $72 500
Monthly Salary (÷12) $6041.67

b)	Shelter	$19 800	$1 650
	Food	23 760	1 980
	Clothing	7 920	660
	Transportation	9 504	792
	Health & Personal Care	6 336	528
	Recreation	4 752	396
	Insurance	3 960	330
	Savings	3 168	264
	Total	$79 200	$6 600

Annual Salary $79 200
Monthly Salary (÷12) $6 600

c)	Shelter	$ 5 450	$ 454.17
	Food	6 540	545.00
	Clothing	2 180	181.67
	Transportation	2 616	218.00
	Health & Personal Care	1 744	145.33
	Recreation	1 308	109.00
	Insurance	1 090	90.83
	Savings	872	72.67
	Total	$21 800	$1 816.67

Annual Salary $21 800
Monthly Salary (÷12) $1816.67

d)	Shelter	$ 2 525	$210.42
	Food	3 030	252.50
	Clothing	1 010	84.17
	Transportation	1 212	101.00
	Health & Personal Care	808	67.33
	Recreation	606	50.50
	Insurance	505	42.08
	Savings	404	33.67
	Total	$10 100	$841.67

Annual Salary $10 100
Monthly Salary (÷12) $841.67

2.

Food	$ 313.46
Household Operation	85.49
Clothing	71.24
Shelter	427.44
Transportation	213.72
Personal Needs	102.59
Savings	142.48
Miscellaneous	68.39
	$1424.81

4-a 1. Statement of Monthly Income & Expenses
for: Dean Charles Family

Income:		
Salary: 4 x 445.25	$1781.00	
Family Allowance 3 x 26.93	80.79	
Total Income		$1861.73
Expenses:		
Mortgage	$ 525.00	
Car payments	213.50	
Food: 4 x 125	500.00	
Property taxes: 725 ÷ 12	60.42	
Life Insurance: 60 ÷ 3	20.00	
Utilities	80.00	
Telephone	10.40	
Total Expenses		$1409.32
Amount Available for savings and other expenditures		$ 452.41

2. Statement of Monthly Income & Expenses
for: Glen & Sara Jones

Income:		
Salary: 1375 + 825	$2200.00	
Total Income		$2200.00
Expenses:		
Mortgage	$ 485.00	
Property Taxes: 1475 ÷ 12	122.92	
Food: 3600 ÷ 12	300.00	
Insurance: 8.10 x 4	32.40	
Utilities	172.50	
Total Expenses		$1112.82
Amount Available for savings and other expenditures		$1087.18

3. Statement of Monthly Income & Expenses
for: Charlie & Bonny Wood Family

Income:		
Salary: (Charlie 4 x 491.82)	$1967.28	
(Bonny 4 x 281.42)	1125.68	
Family Allowance	107.72	
Total Income		$3200.68
Expenses:		
1st Mortgage	$ 531.50	
2nd Mortgage	201.65	
Car payments	237.75	
Boat payments (52.75 x 4)	211.00	
Property taxes (1965.40 ÷ 12)	163.78	
Food (185 x 4)	740.00	
Insurance Life	30.20	
Homeowner's Insurance	248.70	
Utilities	407.60	
Telephone	21.60	
Total Expenses		$2793.78
Amount Available for savings and other expenditures		$ 406.90

4. a) Statement of Yearly Income & Expenses
for: Dean Charles Family

Income:		
Salary (445.25 x 52)	$23 153.00	
Family Allowance (80.79 x 12)	969.48	
Total Income		$24 122.48
Expenses:		
Mortgage (525 x 12)	$ 6 300.00	
Car payments (213.50 x 12)	3 416.00	
Food (125 x 52)	6 500.00	
Property taxes	725.00	
Life Insurance (60 x 4)	240.00	
Utilities (80 x 12)	960.00	
Telephone (10.40 x 12)	124.80	
Total Expenses		$18 265.00
Amount Available for savings and other expenditures		$ 5 856.68

b) Statement of Yearly Income & Expenses
for: Glen & Sara Jones

Income:		
Salary	$26 400.00	
Total Income		$26 400
Expenses:		
Mortgage	$ 5 820.00	
Property taxes	1 475.00	
Food	3 600.00	
Insurance	421.20	
Utilities	2 070.00	
Total Expenses		$13 386.20
Amount Available for savings and other expenditures		$13 013.80

c) Statement of Yearly Income & Expenses
for: Charlie & Bonny Wood Family

Income:		
Salary	$40 208.48	
Family Allowance	1 292.64	
Total Income		$41 501.12
Expenses:		
1st Mortgage	$ 6 378.00	
2nd Mortgage	2 419.80	
Car payments	2 853.00	
Boat payments	2 743.00	
Property taxes	1 965.40	
Food	8 880.00	
Utilities	4 891.20	
Telephone	259.20	
Total Expenses		$30 389.60
Amount Available for savings and other expenditures		$11 111.52

4-c 1. a) Statement of Yearly Amount Left for Further Purchases and Savings
for: Dean Charles Family

Total Available Income:		$5856.68
Estimated additional expenses per year:		
Clothing	$ 725.00	
Entertainment	250.00	
Christmas	230.00	
Newspaper & Periodicals	65.80	
Car Maintenance	630.00	
Gas & Oil (20 x 52)	1040.00	
Total Estimated Expenses		$2940.80
Amount left for further purchases and savings		$2915.88

b) Monthly amount available for additional purchases and savings:

$2915.88 ÷ 12 = $242.99

2. a) Statement of Monthly Income & Expenses
for: Erika & Tom Elsimatesky Family

Income:		
Salary: Erika	$ 806.40	
Tom	1642.00	
Family Allowance	53.86	
Total Income		$2502.26
Expenses:		
Mortgage	$ 725.60	
Car payments	186.40	
Property taxes	153.35	
Insurance	25.68	
Utilities	311.60	
Telephone	11.30	
Food	540.00	
Total Expenses		$1953.93
Amount Available for savings and other expenditures		$ 548.33

b) Statement of Yearly Income & Expenses
for: Erika & Tom Elsimatesky Family

Income:		
Salary: Erika	$10 483.20	
Tom	21 346.00	
Family Allowance	646.32	
Total Income		$32 475.52
Expenses:		
Mortgage	$ 8 707.20	
Car payments	2 236.80	
Property taxes	1 840.16	
Insurance	308.10	
Utilities	3 739.20	
Telephone	135.60	
Food	7 020.00	
Total Expenses		$23 987.06
Amount Available for savings and other expenditures		$ 8 488.46

c) Statement of Yearly Amount Left for Further Purchases and Savings
for: Erika & Tom Elsimatesky Family

Total Available Income:		$8488.46
Estimated additional expenses per year:		
Clothing	$ 650.00	
Entertainment	180.00	
Vacation	470.00	
Car maintenance	560.00	
Special holidays	240.00	
Babysitting	2548.00	
Magazine subscriptions	42.00	
Total Estimated Expenses		$4690.00
Amount left for further purchases and savings		$3798.46

d) Monthly amount available for additional purchases and savings:

$3798.46 ÷ 12 = $316.54

5 1. 4-a-1. $445.25 x 8 = $3562.00 reserve fund
2. $4400 reserve fund
3. $6185.92 reserve fund

4-c-2. $4896.80 reserve fund

2. Substitute amounts on Budget Form for questions as required.

Key example for 4-a-1 only

Dean Charles Family

BUDGET FORM

STEP 1: INCOME

ANNUAL TAKE-HOME PAY $23,153.00 NET INCOME FOR THE MONTH $2,010.21

OTHER INCOME $969.48

ANNUAL NET INCOME $24,122.48

STEP 2: FIXED EXPENSES

	J	F	M	A	M	J	J	A	S	O	N	D
RENT (MORTGAGE)	$525.	$525	$525	$525	$525.	$525	$525.	$525	$525	$525	$525	$525
UTILITIES	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0
TELEPHONE	10.46	10.46	10.46	10.46	10.46	10.46	10.46	10.46	10.46	10.46	10.46	10.46
CAR DEBT	213.50	213.50	213.50	213.50	213.50	213.50	213.50	213.50	213.50	213.50	213.50	213.50
CREDIT CARD DEBT												
OTHER DEBT												
CAR INS. PREMIUM												
LIFE INS. PREMIUM	20	20	20	20	20	20	20	20	20	20	20	20
CAR MAINTENANCE	52.50	52.50	52.50	52.50	52.50	52.50	52.50	52.50	52.50	52.50	52.50	52.50
MISCELLANEOUS (PROPERTY TAXES)	60.42	60.42	60.42	60.42	60.42	60.42	60.42	60.42	60.42	60.42	60.42	60.42

TOTAL FOR THE YEAR $12,396.80

*SET ASIDE FOR THE MONTH $1,032.98

STEP 3: LIVING ALLOWANCES FOR A MONTH

GROCERIES	$500.00	ENTERTAINMENT	$20.83
LAUNDRY		PERSONAL (Donations)	$19.17
CLOTHING	$60.42	OTHER (Subscriptions)	$5.48
GASOLINE	$86.67		

TOTAL LIVING ALLOWANCE FOR A MONTH $692.57

STEP 4: RESERVE FUND

RESERVE FUND (TWO MONTHS' PAY) WILL GROW UNTIL IT REACHES $3,562.00

RESERVE FUND PAYMENTS FOR THE MONTH $296.83

STEP 5: BUDGET BALANCE SHEET

NET INCOME FOR A MONTH (STEP 1)	$2,010.21
"SET ASIDE" FOR A MONTH (STEP 2)	$1,032.98
LIVING ALLOWANCE FOR A MONTH (STEP 3)	$692.57
RESERVE FUND PAYMENTS FOR A MONTH (STEP 4)	$296.83
SAVINGS FOR A MONTH (TOTAL 2, 3, 4, AND DEDUCT FROM 1)	$ 0

* note: Steps 2, 3 & 4 total $2022.88. Since net income is $2010; the reserve fund would have to be cut back to balance the budget (p. 17)

Glen & Sara Jones

BUDGET FORM

STEP 1: INCOME

ANNUAL TAKE-HOME PAY $26,400 NET INCOME FOR THE MONTH $2200!

OTHER INCOME

ANNUAL NET INCOME $26,400

STEP 2: FIXED EXPENSES

	J	F	M	A	M	J	J	A	S	O	N	D
RENT												
UTILITIES												
TELEPHONE												
CAR DEBT												
CREDIT CARD DEBT												
OTHER DEBT												
CAR INS. PREMIUM												
LIFE INS. PREMIUM												
CAR MAINTENANCE												
MISCELLANEOUS												

TOTAL FOR THE YEAR $9,786.20

"SET ASIDE" FOR THE MONTH $815.52

STEP 3: LIVING ALLOWANCES FOR A MONTH

GROCERIES	$300.	ENTERTAINMENT	
LAUNDRY		PERSONAL	
CLOTHING		OTHER	
GASOLINE			

TOTAL LIVING ALLOWANCE FOR A MONTH $300.

STEP 4: RESERVE FUND

RESERVE FUND (TWO MONTHS' PAY) WILL GROW UNTIL IT REACHES $4400

RESERVE FUND PAYMENTS FOR THE MONTH $366.67

STEP 5: BUDGET BALANCE SHEET

NET INCOME FOR A MONTH (STEP 1)	$2,200.
"SET ASIDE" FOR A MONTH (STEP 2)	$815.52
LIVING ALLOWANCE FOR A MONTH (STEP 3)	$300.00
RESERVE FUND PAYMENTS FOR A MONTH (STEP 4)	$366.67
SAVINGS FOR A MONTH (TOTAL 2, 3, 4, AND DEDUCT FROM 1)	$717.81

Lakeslie & Bonny Wood

BUDGET FORM

STEP 1: INCOME

ANNUAL TAKE-HOME PAY $40,208.48 NET INCOME FOR THE MONTH $3458.43

OTHER INCOME $1,292.64

ANNUAL NET INCOME $41,501.12

STEP 2: FIXED EXPENSES

	J	F	M	A	M	J	J	A	S	O	N	D
RENT												
UTILITIES												
TELEPHONE												
CAR DEBT												
CREDIT CARD DEBT												
OTHER DEBT												
CAR INS. PREMIUM												
LIFE INS. PREMIUM												
CAR MAINTENANCE												
MISCELLANEOUS												

TOTAL FOR THE YEAR $21,509.60

*SET ASIDE FOR THE MONTH $1,792.47

STEP 3: LIVING ALLOWANCES FOR A MONTH

GROCERIES	$740.00	ENTERTAINMENT	
LAUNDRY		PERSONAL	
CLOTHING		OTHER	
GASOLINE			

TOTAL LIVING ALLOWANCE FOR A MONTH $740

STEP 4: RESERVE FUND

RESERVE FUND (TWO MONTHS' PAY) WILL GROW UNTIL IT REACHES $6,185.92

RESERVE FUND PAYMENTS FOR THE MONTH $515.49

STEP 5: BUDGET BALANCE SHEET

NET INCOME FOR A MONTH (STEP 1)	$3,458.43
"SET ASIDE" FOR A MONTH (STEP 2)	$1,792.47
LIVING ALLOWANCE FOR A MONTH (STEP 3)	$740.00
RESERVE FUND PAYMENTS FOR A MONTH (STEP 4)	$515.49
SAVINGS FOR A MONTH (TOTAL 2, 3, 4, AND DEDUCT FROM 1)	$410.47

Review

1. Statement of Monthly Income & Expenses
for: Elaine Soluk

Income:		
Salary	$880.00	
Total Income		$880.00
Expenses:		
Apartment rent	$140.00	
Food	96.00	
Telephone	2.20	
Laundry	5.60	
Lunch expense	46.00	
Transportation	26.00	
Total Expenses		$315.80
Amount Available for savings and other expenses		$564.20

2. a)

	Annual	Monthly
Shelter	$ 9 075.00	$ 756.25
Food	10 890.00	907.50
Clothing	3 630.00	302.50
Transportation	4 356.00	363.00
Health & Personal Care	2 904.00	242.00
Recreation	2 178.00	181.50
Insurance	1 815.00	151.25
Savings	1 452.00	121.00
Total	$36 300.00	$3025.00

b)

	Annual	Monthly
Shelter	$ 5 850.00	$ 487.50
Food	7 020.00	585.00
Clothing	2 340.00	195.00
Transportation	2 808.00	234.00
Health & Personal Care	1 872.00	156.00
Recreation	1 404.00	117.00
Insurance	1 170.00	97.50
Savings	936.00	78.00
Total	$23 400.00	$1950.00

c)	Shelter	$12 050.00	$1004.17
	Food	14 460.00	1205.00
	Clothing	4 820.00	401.67
	Transportation	5 784.00	482.00
	Health & Personal Care	3 856.00	321.33
	Recreation	2 892.00	241.00
	Insurance	2 410.00	200.83
	Savings	1 928.00	160.67
	Savings	$48 200.00	$4016.67

3. a) Statement of Monthly Income & Expenses
for: The Dennis Murray Family

Income:		
Salary: Dennis Murray	$933.33	
Mrs. Murray	850.24	
Family Allowance	26.93	
Total Income		$1810.50
Expenses:		
Mortgage payment	$ 425.00	
Personal loan	165.00	
Property taxes	60.83	
Heat	61.67	
Electricity & Water	41.75	
Telephone	10.50	
House Insurance	20.17	
Food	420.00	
Furniture Loan	212.00	
Car expenses	180.00	
Auto Insurance	24.00	
Total Expenses		$1620.92
Amount Available for savings and other expenditures		$ 189.58

b) Statement of Yearly Income & Expenses
for: Dennis Murray Family

Income:		
Salary	$22 253.08	
Family Allowance	323.16	
Total Income		$22 576.24
Expenses:		
Mortgage	$ 5 100.00	
Personal Loan	1 980.00	
Property taxes	730.00	
Heat	740.00	
Electricity & Water	501.00	
Telephone	126.00	
House Insurance	242.00	
Food	5 460.00	
Furniture Loan	2 544.00	
Car Expenses	2 340.00	
Auto Insurance	288.00	
Total Expenses		$20 051.00
Amount Available for savings and other expenditures		$ 2 525.24

c) Statement of Yearly Amount Left for Further Purchases and Savings
for: Dennis Murray Family

Total Available Income		$2525.24
Estimated additional expenses per year:		
Clothing	$795.00	
Entertainment	330.00	
Christmas	135.00	
Newspapers, etc.	125.00	
Further car expenses	255.00	
Vacation	850.00	
Total Estimated Expenses		$2490.00
Amount left for further purchases and savings		$ 35.24

d) Monthly amount available for additional purchases and savings:

35.24 ÷ 12 = $2.94

4. #1 - Reserve fund $1760.00
   #3 - Reserve fund $3567.14

5. Must complete your own Budget Balance Sheet

Dennis Murray Family

BUDGET FORM

STEP 1: INCOME

ANNUAL TAKE-HOME PAY $22,253.88 NET INCOME FOR THE MONTH $1,881.3

OTHER INCOME 323.16

ANNUAL NET INCOME $22,576.24

STEP 2: FIXED EXPENSES

	J	F	M	A	M	J	J	A	S	O	N	D
RENT												
UTILITIES												
TELEPHONE												
CAR DEBT												
CREDIT CARD DEBT												
OTHER DEBT												
CAR INS. PREMIUM												
LIFE INS. PREMIUM												
CAR MAINTENANCE												
MISCELLANEOUS												

TOTAL FOR THE YEAR $14,591.00

*SET ASIDE FOR THE MONTH $1,215.92

STEP 3: LIVING ALLOWANCES FOR A MONTH

GROCERIES	$155.	ENTERTAINMENT	$27.50
Vacation LAUNDRY	70.00	PERSONAL (Christine)	11.25
CLOTHING	66.25	OTHER (Subscriptions)	10.92
(Car payment) GASOLINE	21.25		

TOTAL LIVING ALLOWANCE FOR A MONTH $661.67

STEP 4: RESERVE FUND

RESERVE FUND (TWO MONTHS' PAY) WILL GROW UNTIL IT REACHES $3567.14

RESERVE FUND PAYMENTS FOR THE MONTH $297.26

STEP 5: BUDGET BALANCE SHEET

NET INCOME FOR A MONTH (STEP 1)	$1,881.35
"SET ASIDE" FOR A MONTH (STEP 2)	$1,215.92
LIVING ALLOWANCES FOR A MONTH (STEP 3)	$ 661.67
RESERVE FUND PAYMENTS FOR A MONTH (STEP 4)	$ 297.26
SAVINGS FOR A MONTH (TOTAL 2, 3, 4, AND DEDUCT FROM 1)	$ —

persons fund to be reduced as Step 2, 3 + 4 exceed net income by ($293.50)

B Taxation

I-1 - Answers will vary depending upon the year

II-1
1. a) 3.3¢ b) 3.6¢ c) 3.7¢
2. a) 8.8% b) 9.1% c) 10.6%
3. a) 44 mills b) 44 mills c) 56 mills

II-2
1. a) $288 b) $1120 c) $3300 d) $9000 e) $2025
2. $1000
3. 7¢ x 15 000 = $980.00 general tax
   93.50 local improvements
   $1073.50 total
4. a) $2300 b) $20 400 c) $960 d) $300
5. 40% x $52 000 = $20 800 assessed value
   5% x $20 800 = $ 1 040 tax bill
6. 30% x $220 000 = $66 000 assessed value
   $66 000 x 5% = $3300 general tax
   $3300 + 150 (1500 ÷ 10) + 86 (860 ÷ 10) = $3536
7. a) $992 b) $3712 c) $190 d) $1650
8.

Cottage	$11 200	$582.40	$582.40
Bungalow	21 840	1201.20	1336.40
Duplex	42 240	2534 40	2703.25
Factory	128 000	7936.00	8281.50

9. $38 000 x 40% = $15 200 assessed value
   $15 200 x 61/100 = $927.20 general tax, + 128.50 = $1055.70 total tax
10. Year 1 - $1020
    Year 2 - $1225 - The amount of general tax increase was 1225 - 1020 = $205

III Hidden Taxes

III-1
1. a) $465.00 b) $516.00 c) $1245.00 d) $1800.00 e) $187.50
2. $121.88

III-2
1. Cost: $250 x 10 = $2500
   Custom tariff: $2500 x 15% = $375

2. Total cost: 100 x $5 = $500
   Custom tariff: $500 x 15% = $75
   Duty-paid value: $500 x $75 = $575.00
   Federal sales tax: $575 x 15% = $86.25
   Total cost to import: $575 + $86.25 = $661.25

3. Tariff from Britain: $17 x 10% = $1.70
   Duty-paid value: $17 + 1.70 = $18.70
   Federal sales tax: $18.70 x 15% = $2.81
   Total cost to import: $18.70 + 2.81 = $21.51

   Tariff from U.S.: $15.50 x 15% = $2.33
   Duty-paid value: $15.50 + 2.33 = $17.83
   Federal sales tax: $17.83 x 15% = $2.67
   Total cost to import: $17.83 + 2.67 = $20.50

   Therefore, U.S. drill is cheaper by $21.51 - 20.50 = $1.01

4. Total cost: 1000 x $3 = $3000
   Custom tariff: $3000 x 15% = $450
   Duty-paid value: $3000 + 450 = $3450
   Federal sales tax: $3450 x 15% = $517.50
   Total cost to import: $3450 + 517.50 = $3967.50

III-3 1. Cost of cards: 100 x $0.50 = $50.00
   Federal sales tax: $50.00 x 15% = $7.50
   Excise tax: 100 x $0.20 = $20.00
   Total cost: $50.00 + $7.50 + $20.00 = $77.50

2. Cost of tobacco: 1500 x $2.50 = $3750.00
   Custom tariff: 1500 x $0.40 = $600.00
   Duty-paid value: $3750 + $600 = $4350
   Federal sales tax: $4350 x 15% = $652.50
   Excise tax: 1500 x $0.90 = $1350.00
   Total imported cost: $4350 + $652.50 + $1350 = $6352.50

3. Cost of watches: $80 x 25 = $2000
   Custom tariff: $2000 x 20% = $400
   Duty-paid value: $2000 + $400 = $2400
   Federal sales tax: $2400 x 15% = $360

   The duty-paid value of each watch is $2400 ÷ 25 = $96;
   only the amount above $50.00 is subject to excise tax.
   Therefore the amount subject to excise tax is $96 - 50 = $46 x 25
   watches = $1150. Excise tax: $1150 x 10% = $115
   Total imported cost: $2400 + $360 + $115 = $2875.00

4. Custom tariff: $1500 x 45% = $675.00
   Duty-paid value: $1500 + $675 = $2175.00
   Federal sales tax: $2175 x 15% = $326.25
   Excise tax: $2175 x 10% = $217.50
   Total imported cost: $2175 + $326.25 + $217.50 = $2718.75

C Insurance

C-2b

1. a) $24.16
   b) $286.50
   c) $139.66
   d) $3947.50
   e) $844.41
   f) $953.08
   g) $571.00
   h) $3386.00
   i) $5577.50
   j) $12 212.32

2. a) $44.59
   b) $88.18
   c) $95.37
   d) $2078.35
   e) $189.49
   f) $197.81

3. $15.40 less
4. $8500.00 1000 x 113.05/13.30
5. $20 000 1000 x 552/27.60
6. $75 335.40
   $105 000

C-3c

1. a) $78
   b) $119
   c) $306
   d) $169
   e) $387
   f) $1020
   g) $44
   h) $537

2. a) $78.00; $139.00; $61.00
   b) $249.00; $558.00; $309.00
   c) $170.00; $327.00; $157.00
   d) $134.00; $243.00; $109.00

3. $1.00 more for Danny McLean

C-3d

1. a) $283.00
   b) $856.00
   c) $483.00
   d) $143.00

2. Homeowner's Policy - 212 + 45 = $257.00
   Tenant's package - 79 + 30 = $109.00
   Annual premium - 257 + 109 = $366.00

3.

dwelling	$140 000
outbuilding	14 000
trees	7 000
contents	56 000
theft	14 000
living expenses	28 000
	$259 000

4. a) Claim $24 000 + 789.00 = $24 789.00

   b) Claim $40 000 + 9000 = $49 000.00
   Note: Fence and garage both covered under 10%, outbuildings which amounts to 10% of 9000. Therefore insurer only receives the $9000.

   c) Claim $8000 + 4300 = $12 300.00

   d) Claim $60 000 + 24 000 = $84 900.00

C-3e

1. a) $45 000, since 80% of 75 000 = $60 000
   b) $5000, since 80% of 35 000 = $28 000
   c) (25 000/0.8 x 38 000) x 19 500 = $16 036.18
   d) $40 000
   e) (20 000/0.8 x 34 000) x 12 500 = $9191.18
   f) (38 000/0.8 x 59 000) x 30 000 = $24 152.54
   g) (25 000/0.8 x 46 000) x 22 000 = $14 945.65
   h) (16 000/0.8 x 26 000) x 5000 = $3846.15

2. $22 000

Unit Review

1. $484.50

2.

Annual	$132.50
Semiannually	67.58
quarterly	34.45
monthly	11.53

3. a) (15 500/1000) x 11.35 = $175.93
   b) 175.93 x 11 = $1935.23

4. (10 500/1000) x 39.86 = $418.53
   (10 500/1000) x 30.65 = $321.83
   difference $ 96.70

5.

dwelling	$ 87 350.00
outbuildings	8 735.00
trees	4 367.50
contents	34 940.00
theft	8 735.00
lifing expense	17 470.00
total coverage	$161 597.50

6. (20 000/1000) x 54.14 = $1082.80 annual pmts
   1082.80 x 0.26 (quarterly) $281.53
   281.53 x 4 = $1126.11 if paid quarterly
   Therefore, saving 1126.11 - 1082.80 = $43.31

7. 80% of 30 000 = $24 000
   The insurance company will pay $24 000 to the insured. (They will not pay more than the face of the policy.)

D. Mortgages

2

1. a) A mortgage is a loan secured by property.
   b) Casey & Evelyn
   c) Bank of Montreal
   d) Mr. Charles
   e) $20 800 ($85 500 - 49 700 = 35 800 - 15 000 = $20 800)
   f) 12 1/2 %
   g) 21%
   h) A second mortgagee takes a greater risk than the first mortgagee because the first mortgagee has first claim on the property if the mortgagor fails to keep up his or her financial commitment.

3a 1.

Schedule of Mortgage Payments

Date	Regular Payment			Unpaid Balance	Owner's Equity
	Principal	Interest	Total		
Feb. 1				$6000	$26,000
1st yr. July 31	750.00	$1080.00	$1830	5250	26750
2nd yr. July 31	750.00	945.00	1695	4500	27500
3rd yr. July 31	750.00	810.00	1560	3750	28250
4th yr. July 31	750.00	675.00	1425	3000	29000
5th yr. July 31	750.00	540.00	1290	2250	29750
6th yr. July 31	750.00	405.00	1155	1500	30500
7th yr. July 31	750.00	270.00	1020	750	31250
8th yr. July 31	750.00	135.00	885	0	32000

2. Schedule of Mortgage Payments

Date	Regular Payment			Unpaid Balance	Owner's Equity
	Principal	Interest	Total		
Mar. 1				$26 000	
Mar. 31	$250	$427.92	$677.92	25750	
Apr. 30	250	423.80	673.80	25500	
May 31	250	419.69	669.69	25250	
June 30	250	415.57	665.58	25000	
July 31	250	411.46	661.46	24750	
Aug. 31	250	407.34	657.34	24500	

a) $2505.78

b) $1500.00

c) 250 per month x 12 = $3000 per year
26 000 ÷ 3000 = 8.7 or 8 years 9 months

3. a)

Schedule of Mortgage Payments

Date	Regular Payment			Unpaid Balance	Owner's Equity
	Principal	Interest	Total		
				$20 000.00	
Year 1 1st pmt	$ 500	$2812.50	$3312.50	49500.00	
2nd pmt	500	2784.38	3284.38	49000.00	
3rd pmt	500	2756.25	3256.25	48500.00	
4th pmt	500	2728.13	3228.13	48000.00	
Year 2 1st pmt	500	2700.00	3200.00	47500.00	
2nd pmt	500	2671.88	3171.88	47000.00	
3rd pmt	500	2643.75	3143.75	46500.00	
4th pmt	500	2615.63	3115.63	46000.00	

b) 5000 - (500 x 4 = 2000 x 10 years)20 000 = $30 000

c) 50 000 ÷ 2000 payments per year = 25 years

4. a)

Schedule of Mortgage Payments

Date	Regular Payment			Unpaid Balance	Owner's Equity
	Principal	Interest	Total		
				$25 000	$20 000
Sep. 15	$320	$390.63	$710.63	24680	20320
Oct 15	320	385.63	705.63	24360	20640
Nov 15	320	380.63	700.63	24040	20960
Dec 15	320	375.63	695.63	23720	21280
Jan 15	320	370.63	690.63	23400	21600
Feb. 15	320	365.63	685.63	23080	21920

b) $2268.78 Interest

c) Principal = 6 x $320 = $1920

d) 25 000/320 x 12 = 6.5 = 6 years 6 months

e) Owner's Equity = $320 x 12 = $3840 per year
$3840 x 5 years = $19 200
$25 000 + $19 200 = $39 200

3b 1.

Schedule of Mortgage Payments

Date	Regular Payment			Unpaid Balance	Owner's Equity
	Principal	Interest	Total		
				$38 000.00	$27000.00
Jan. 15	$59.58	($38000 x .1975 x 1/12) $625.42	$685	37940.42	27059.58
Feb. 15	60.56	624.44	685	37879.86	27120.14
Mar. 15	61.56	623.44	685	37818.30	27181.70
Apr. 15	62.57	622.43	685	37755.73	27244.27
May 15	63.60	621.40	685	37692.13	27307.87
June 15	64.65	620.35	685	37627.48	27372.52

a) Interest $3737.48

b) Principal $372.52

2.

Schedule of Mortgage Payments

Date	Regular Payment			Unpaid Balance	Owner's Equity
	Principal	Interest	Total		
Aug.14				$39200.00	$13800.00
Sep.14	$101.33	$718.67	$820	39098.67	13901.33
Oct.14	103.19	716.81	820	38995.48	14004.52
Nov.14	105.08	714.92	820	38890.40	14109.60
Dec.14	107.01	712.99	820	38783.39	14216.61
Jan.14	108.97	711.03	820	38674.42	14325.58
Feb.14	110.97	709.03	820	38563.45	14436.55
Mar.14	113.00	707.00	820	38450.45	14549.55
Apr.14	115.08	704.93	820	38335.37	14664.63

3b,ii

1. a) 6000/1000 x 17.50 = $105.00
   b) 42 200/1000 x 18.959 = $800.07
   c) 64 500/1000 x 18.910 = $1219.70
   d) 22 000/1000 x 15.978 = $351.52
   e) 95 500/1000 x 15.845 = $1513.20
   f) 120 500/1000 x 9.52 = $1147.16
   g) 73 400/1000 x 19.303 = $1416.84
   h) 80 000/1000 x 19.134 = $1530.72

2. a) Payment = 42 000/1000 x 17.906 = $752.05

   b)

Schedule of Mortgage Payments

Date	Regular Payment			Unpaid Balance	Owner's Equity
	Principal	Interest	Total		
				$42 000.00	$30 000.00
Aug. 1	$104.55	$647.50	$752.05	41895.45	30104.55
Sep. 1	106.16	645.89	752.05	41789.29	30210.71
Oct. 1	107.80	644.25	752.05	41681.49	30318.51
Nov. 1	109.46	642.59	752.05	41572.03	30427.97
Dec. 1	111.15	640.90	752.05	41460.88	30539.12
Jan. 1	112.86	639.19	752.05	41348.02	30651.98
Feb. 1	114.60	637.45	752.05	41233.42	30766.58

Mr. Mortgage Buyer
Statement of Adjustments
19-- 04 15

Particulars	Charged to Buyer	Credited to Buyer
Purchase Price	$31 000.00	
Deposit		$ 1 000.00
Mortgage		16 000.00
Interest on Mortgage (March 31-April 15)		118.36
Taxes April 15 to December 31	1 090.83	
Insurance for 30 months	316.67	
Water bill February 28 to April 15		20.40
Balance due seller		15 268.74
	$32 407.50	$32 407.50

Explanation

b) purchase price of $31 000 charged to buyer
c) buyer is credited with the $1000 deposit
d) the buyer is assuming the existing mortgage at 18%. The unpaid balance on April 15 is $16 000. The seller pays the interest on the mortgage from the time of the last monthly payment to the closing date because he owned the house during this period.
Interest from March 31 to April 15:
16 000 x 0.18 x 15/365 = $118.36
Interest is credited to the buyer. The unpaid balance of the mortgage is credited to the buyer since he will pay it in the future.

3. a) 15 000/1000 x 16.648 = $249.72
   b) 15 000 x .195 x 1/12 = $243.75

4. a) 17 000/1000 x 19.111 = $324.89
   b) 17 000 x .205 x 1/12 = $290.42
   c) Payment = 10 x 324.89 x 12 = $38 986.80
   d) Interest = $38 986.80 - 17 000 = $21 986.80

5. a) 30 000/1000 x 10.32 = $309.60
   b) $104 880
      309.60 x 12 = 3715.20 per year
      3715.20 x 25 years - 92 880
      add down payment 12 000
      104 880

6. a) 55 000/1000 x 10.07 - $553.85
   b) 8000/1000 x 17.055 = $144.44
   c) $553.85 x 12 x 30 = $119 386.00
      $144.44 x 12 x 10 = 17 332.80
      Down payment 20 000.00
      Total payment $236 718.80

e) Taxes of $1540 have been paid in full. The new owner pays for the taxes from April 15 to December 31 or 8.5 months.
Taxes for 8.5 months - $1540 x 8.5/12 = $1090.83 (charge to buyer)

f) The three year premium was paid on October 15 of the previous year. Since the policy was in effect for 6 months before the closing date, the buyer should pay for 30 months insurance.
Cost of 30 months insurance = 380 x 30/36 = $316.67 (charge to buyer)

g) The last quarterly water bill for $40.80 was paid on February 28. The seller pays the water expense from February 28 to April 15, therefore, one-half of the bill is credited to the buyer.

2.

Mr. Mortgage Buyer

Statement of Adjustments
19-- 07 31

Particulars	Charged to Buyer	Credited to Buyer
Purchase price	$36 800.00	
Mortgage		$22 750.00
Mortgage interest		369.69
Taxes (12.4 x 62 x 1/2)		384.40
Insurance (320 x 1/2)	160.00	
Water		36.20
Deposit		1 500.00
Balance Due Seller		11 919.71
	$36 960.00	$36 960.00

2.

Mr. Mortgage Buyer
Statement of Adjustments
19-- 11 30

Particulars	Charged to Buyer	Credited to Buyer
Purchase Price	$86 000.00	
Deposit		$ 5 000.00
Taxes (34 x 52 ÷ 12)	147.33	
Mortgage		62 000.00
Mortgage interest		930.00
Oil (188 x .312)	58.61	
Insurance (490 x 1/2)	245.00	
Water (53.40 x 2/3)		35.60
Second Mortgage		18 000.00
Balance Due Seller		485.34
	$86 450.94	$86 450.94

Review Exercises

1.

Schedule of Mortgage Payments

Date	Regular Payment			Unpaid Balance	Owner's Equity
	Principal	Interest	Total		
				$13000	$25000
June 1	$1000	$650	$1650	12000	26000
Dec. 1	1000	600	1600	11000	27000
June 1	1000	550	1550	10000	28000
Dec. 1	1000	500	1500	9000	29000

2. a)

Schedule of Mortgage Payments

Date	Regular Payment			Unpaid Balance	Owner's Equity
	Principal	Interest	Total		
				$29 000	$19000
1st quarter	$450	$815.63	$1265.63	28550	19450
2nd quarter	450	802.97	1252.97	28100	19900
3rd quarter	450	790.31	1240.31	27650	20350
4th quarter	450	777.66	1227.66	27200	20800

b) Amount = \$29 000 - (\$450 x 5 x 4) = 29 000 - 9000 = \$20 000

c) Time = \$29 000/450 x 4 = 16.11 years = 16 years, 3 months

3. a) $25 000/1000 x 10.16 = $254

b)

Schedule of Mortgage Payments

Date	Regular Payment			Unpaid Balance	Owner's Equity
	Principal	Interest	Total		
Sep.				$25 000.00	$20 000.00
Oct.	$24.83	$229.17	$254.00	24975.17	20024.83
Nov.	25.06	228.94	254.00	24950.11	20049.89
Dec.	25.29	228.71	254.00	24924.82	20075.18
Jan.	25.52	228.48	254.00	24899.30	20100.70

4. 118.95 ÷ 15 = 7.93
   7.93 is in the 9% column

5.

Mr. Mortgage Buyer
Statement of Adjustments
19-- 03 15

Particulars	Charged to Buyer	Credited to Buyer
Purchase Price	$53 000.00	
Mortgage		$32 850.00
Deposit		1 000.00
Interest on mortgage (32 850 x 0.1/12 x 0.5)		136.88
Taxes ($15 000 x 0.0683 x 3.2/12)		213.44
Fire Insurance	130.00	
Water ($28.75 x 2/3)		19.17
Balance Due Seller		18 910.51
	$53 130.00	$53 130.00

6.

Mr. Mortgage Buyer
Statement of Adjustments
19-- 05 01

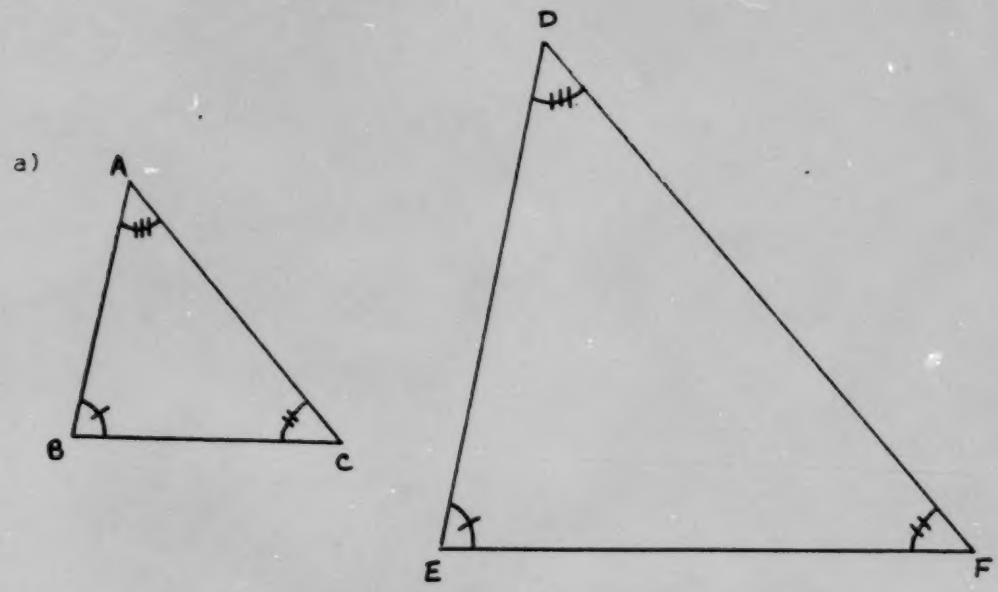
Particulars	Charged to Buyer	Credited to Buyer
Purchase Price	$43 600.00	
Deposit		$ 950.00
Mortgage		30 675.00
Interest on mortgage ($30 675 x 0.105 x 15/365)		132.37
Taxes (18 000/1000 x 58.2 x 1/3)	349.20	
Insurance (30 000/100 x 1.55 x 5/12)	193.75	
Oil (300 x .297)	89.10	
Balance Due Seller		12 474.68
	$44 232.05	$44 232.05

C.III. TRIGONOMETRY (1 module)

NOTE: Express all trigonometric ratios correct to five decimal places.

1,2 1. For each of the pairs of similar triangles, identify the corresponding parts.

a)



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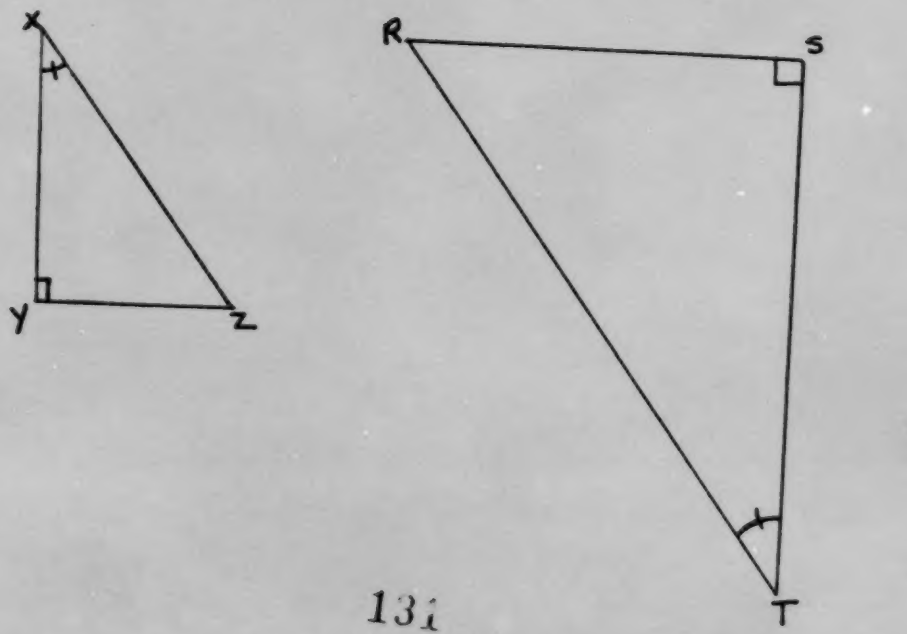

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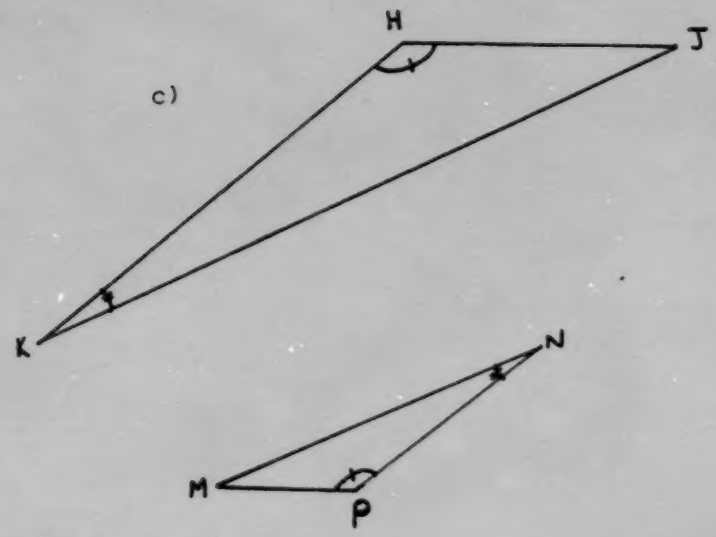

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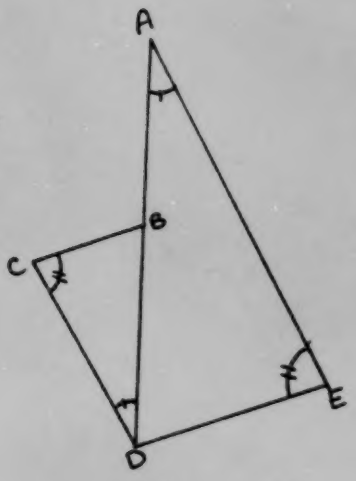

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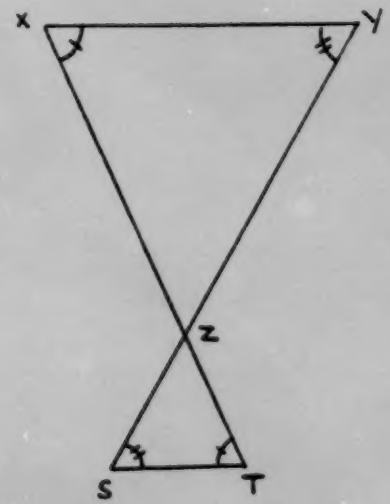
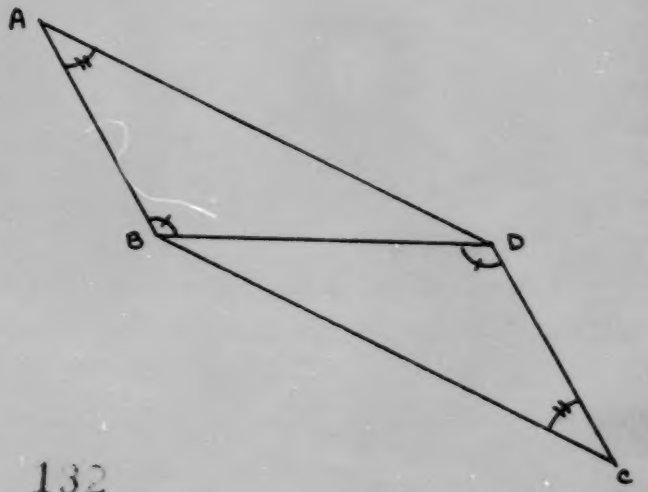
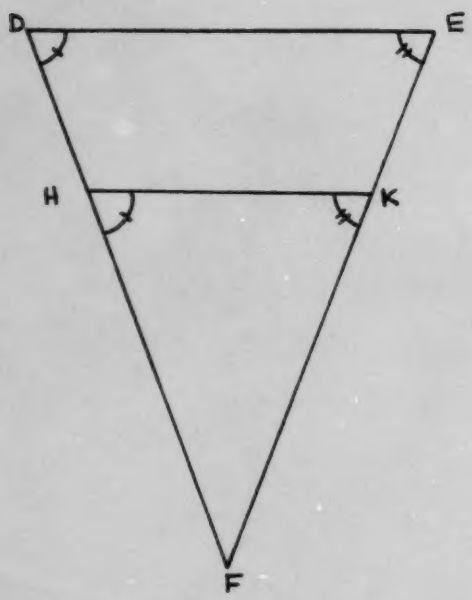


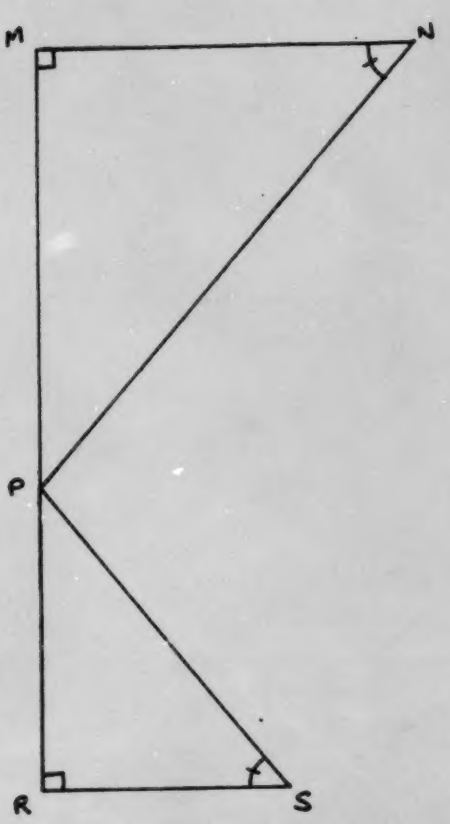
f)



g)

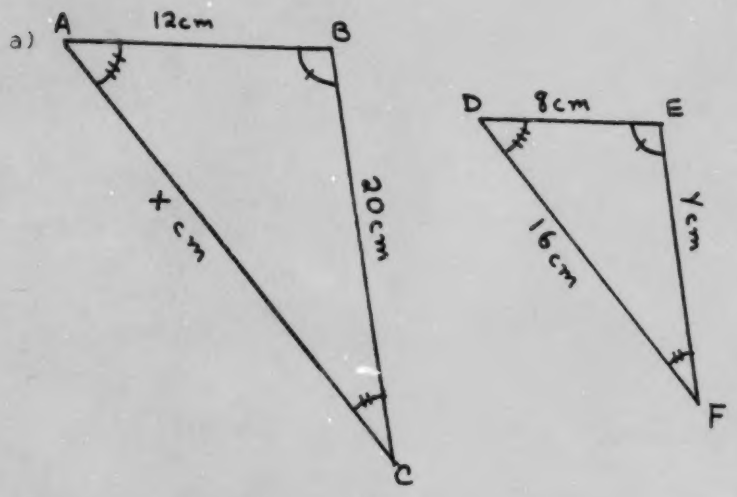


h)



3 1. For each of the pairs of similar triangles, solve for x and y, using equivalent ratios.

a)



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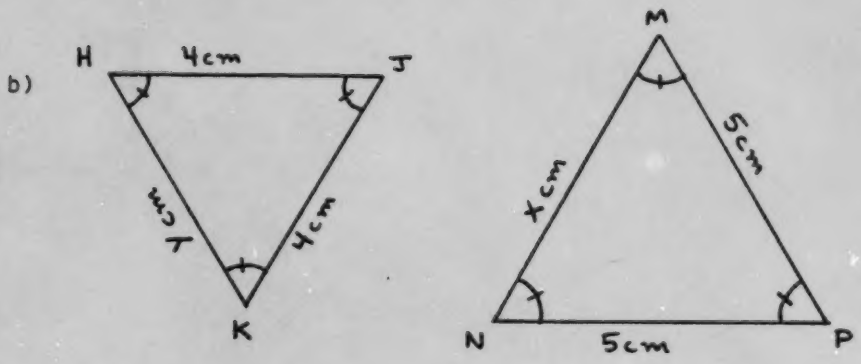
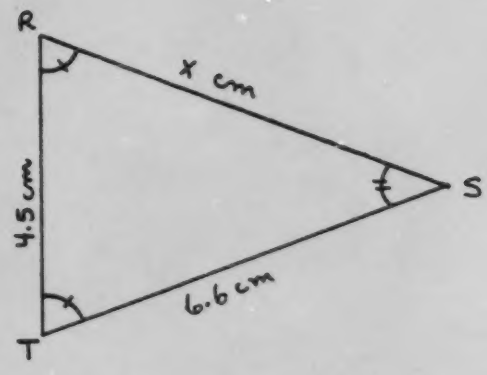
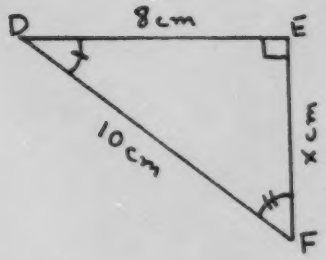
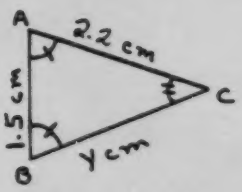


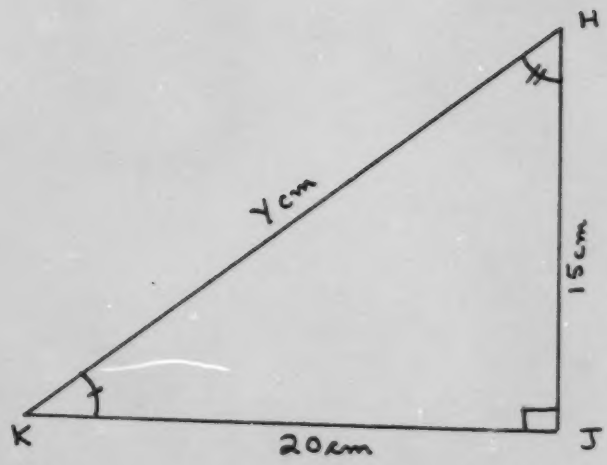
c)


R
x cm
S
4.5 cm
6.6 cm
T

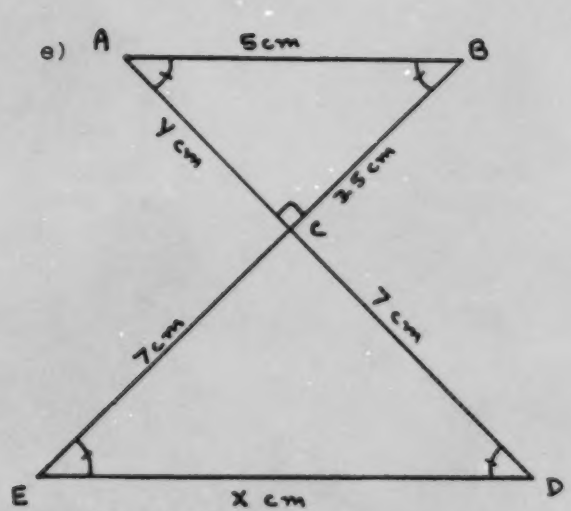
d)


D
8 cm
E
10 cm
x cm
F

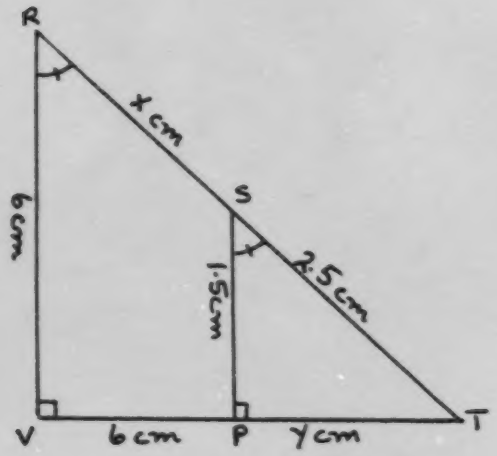

A
2.2 cm
C
1.5 cm
B
y cm


H
y cm
15 cm
K
20 cm
J

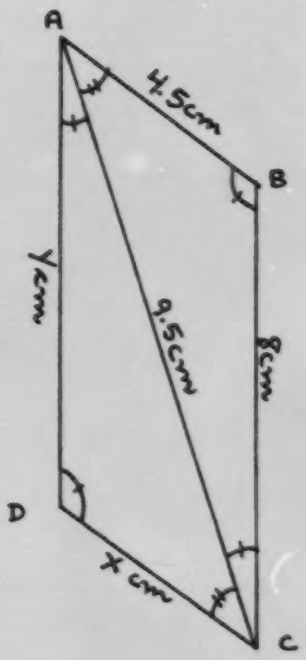
e)


A
5 cm
B
y cm
2.5 cm
C
7 cm
7 cm
E
x cm
D

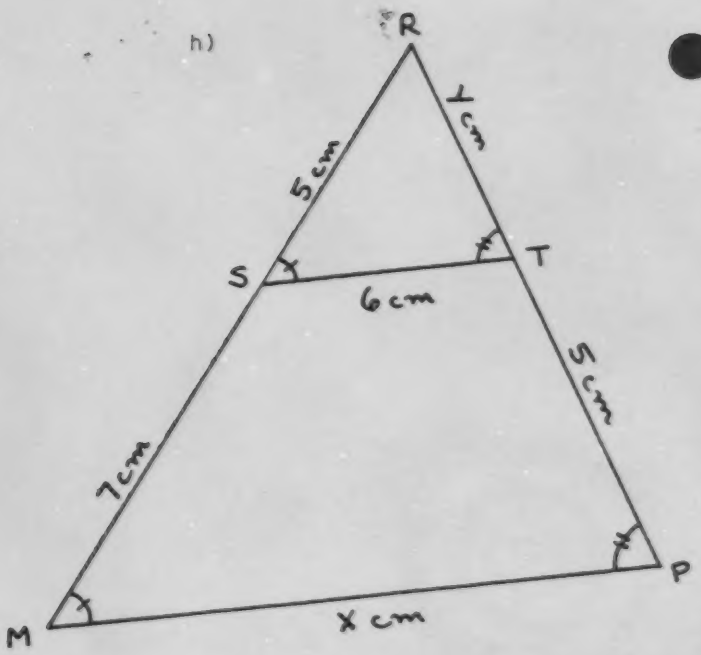
f)


R
x cm
S
6cm
1.5cm
2.5cm
V
6 cm
P
y cm
T

g)


A
4.5cm
B
y cm
9.5cm
8cm
D
x cm
C

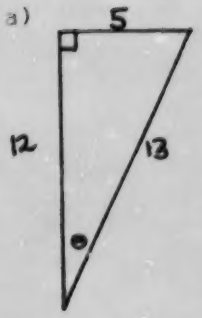
h)


R
y cm
5cm
S
T
6 cm
5 cm
7cm
M
x cm
P

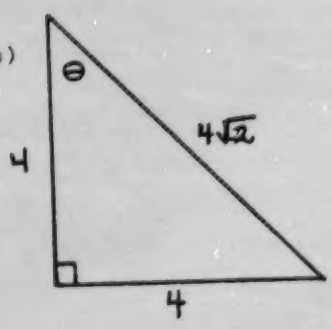
4 1. Copy the table below. Use the following diagrams to complete the table.

	Hypotenuse	Side Opposite to θ	Side Adjacent to θ	Sin θ	Cos θ	Tan θ
a)						
b)						
c)						
d)						
e)						

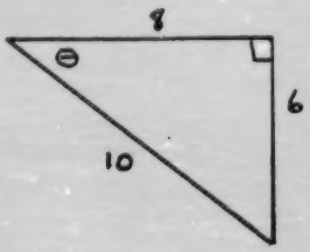
a)



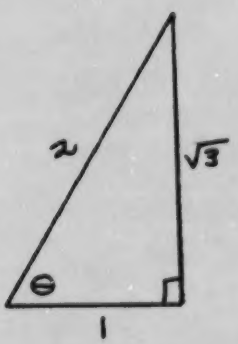
b)



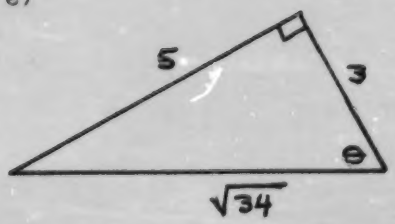
c)



d)

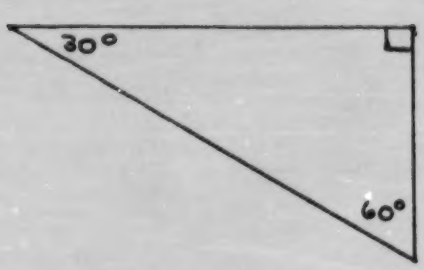


e)

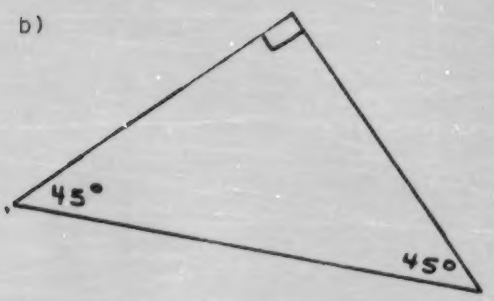


2.

a)



b)



i) Find the length of each side of the above triangles (by measuring), and write them on the diagram.

ii) Using the above information, complete the following table. (Write answers as decimals.)

Sin 60° =	Sin 30° =	Sin 45° =
Cos 60° =	Cos 30° =	Cos 45° =
Tan 60° =	Tan 30° =	Tan 45° =

5 I. Use a calculator to find the numerical value of each of the following:

a) Sin 60° =
b) Cos 60° =
c) Tan 60° =
d) Sin 30° =
e) Cos 30° =
f) Tan 30° =
g) Sin 45° =
h) Cos 45° =
i) Tan 45° =

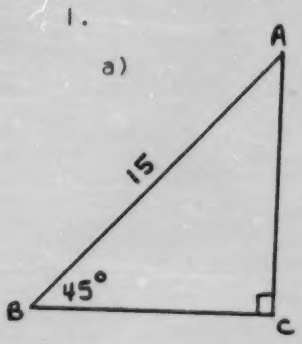
Note: These answers should compare with the answers from the preceding table.

2. Use a calculator to find the numerical value of each of the following:

a) Sin 10° =
b) Cos 23° =
c) Tan 47° =
d) Sin 81° =
e) Cos 16° =
f) Tan 59° =
g) Sin 50° =
h) Cos 72° =
i) Tan 2° =
j) Sin 63° =
k) Cos 89° =
l) Tan 31° =

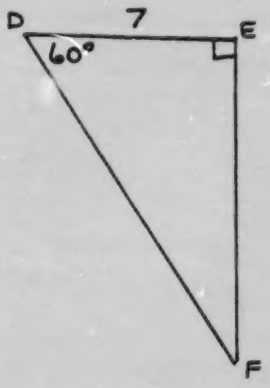
6 1.

a)



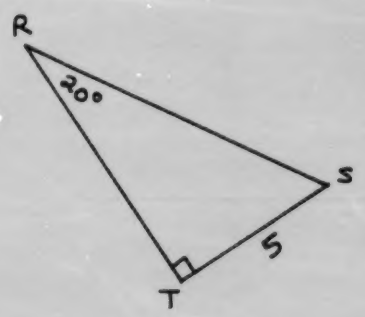
Find LA,b

b)



Find LF,e

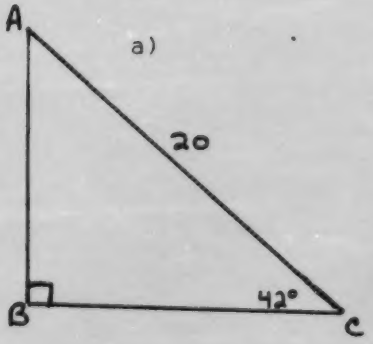
c)



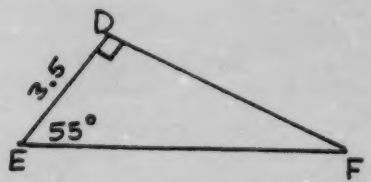
Find t,s

2. Solve the following triangles.

a)



b)



c) ΔABC, where $\angle A = 90^\circ$, $\angle C = 39^\circ$, $c = 18$

d) ΔDEF, where $\angle E = 90^\circ$, $\angle D = 15^\circ$, $e = 16.2$

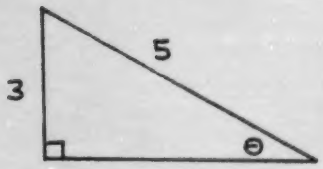
3. From a point A, 27 m from the base of a flag pole, the angle of elevation of the top of the pole is 57°. What is the height of the pole?

4. From the roof of a building, the angle of depression of a car parked on the street is 65°. The building is 45 m high. If the base of the building is in the same plane as the street, how far from the base of the building is the car?

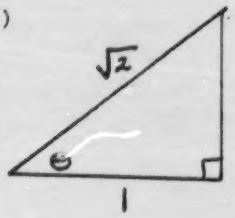
5. A ramp is inclined at an angle of 20° to the horizontal. A man walks down the ramp a distance of 40 m. How many metres below his starting point is the man?

7 1. Determine θ in the following right triangles.

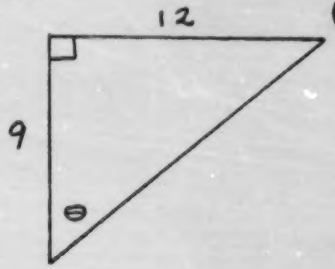
a)



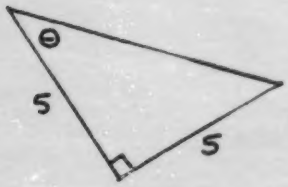
b)



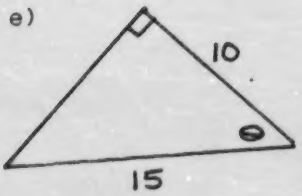
c)



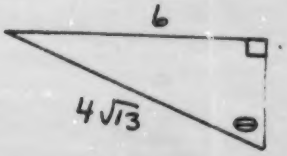
d)



e)



f)

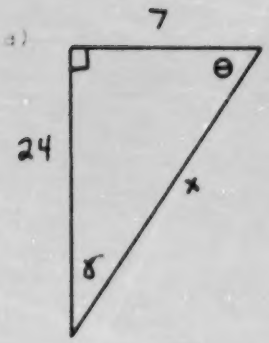


2. Determine the value of θ ($0 \leq \theta \leq 90$) given the following:

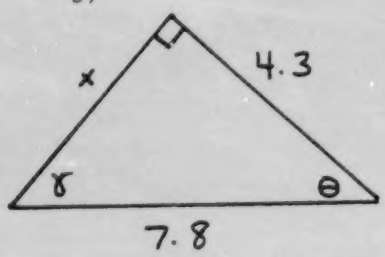
a) Sin θ = .25882
b) Cos θ = .30902
c) Tan θ = 1.42815
d) Sin θ = .58779
e) Cos θ = .88295
f) Tan θ = 1.00000
g) Sin θ = .87462
h) Cos θ = .99863
i) Tan θ = 28.63625

8 1. Determine x, θ, and γ for the following right triangles.

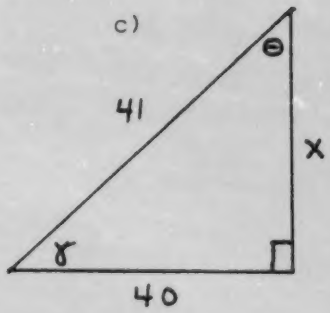
a)



b)



c)



2. Solve the following right triangles.

a) ΔABC, $\angle A = 90^\circ$, a = 9.7, b = 6.5
b) ΔDEF, $\angle F = 90^\circ$, d = 14, e = 10
c) ΔRST, $\angle S = 90^\circ$, r = 33.67, s = 48.25

3. For a flag pole 32 m in height to cast a shadow 60 m in length, what would the angle of elevation of the sun have to be?

4. A 10 m ladder is placed against a building so that the top of the ladder just reaches the eave on a building. If the distance from the eave to the base of the building is 8.2 m:

a) What angle does the ladder make with the ground?
b) How far is the base of the ladder from the base of the building?

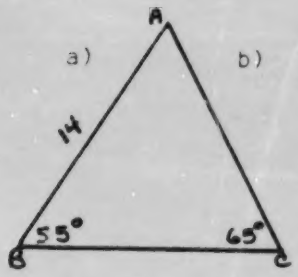
5. Given: rectangle ABCD where AB = 6.5 cm and BC = 7.2 cm, find:

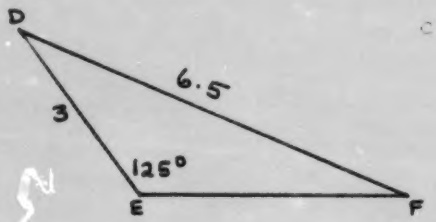
a) the length of diagonal AC;
b) the measure of the angle formed by the diagonal AC and side AB.

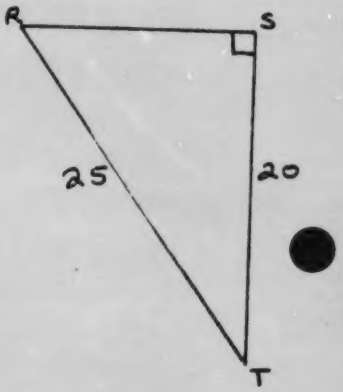
9 1. Determine the numerical value for the following ($90 \leq \theta \leq 180$):

a) Sin 100° =
b) Cos 120° =
c) Tan 150° =
d) Sin 135° =
e) Cos 170° =
f) Tan 95° =
g) Sin 165° =
h) Cos 180° =
i) Tan 117° =
j) Sin 90° =

10 1. Use the Law of Sines to solve the following triangles:



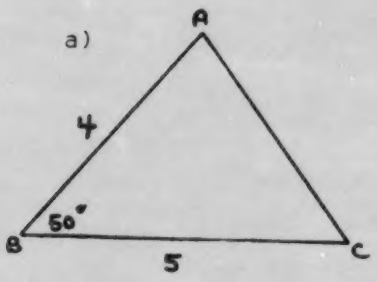




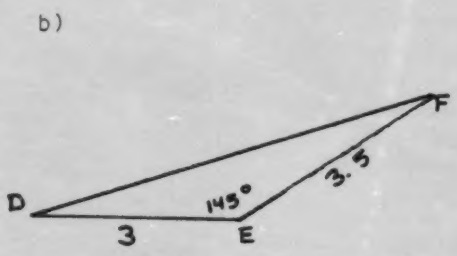
2. Use the Law of Sines for the following:

a) Given ΔXYZ, $\angle X = 40^\circ$, $\angle Y = 70^\circ$, YZ = 12, find XY
b) Given ΔMNP, $\angle N = 90^\circ$, $\angle P = 50^\circ$, MN = 8, find MP
c) Given ΔHJK, $\angle J = 150^\circ$, HK = 10, JK = 5.2, find $\angle H$

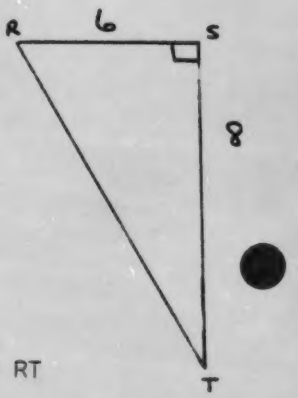
11 1. Use the Law of Cosines for the following:



Find AC



Find DE 12



Find RT

d) ΔHJK, HJ = 9, HK = 12, JK = 13, Find $\angle H$

e) ΔXYZ, XY = 8, YZ = 3, XZ = 8, Find $\angle X$

f) ΔMNP, MN = 2.5, MP = 7.3, NP = 5, Find $\angle N$

12 Solve the following problems which require the use of Trig Ratios, the Law of Sines or the Law of Cosines.

1. Two fire towers are situated 15 km apart on an east-west line. The fire warden in the west tower sees a fire at 30° east of north, the fire warden in the east tower sees the same fire 15° west of north. How far is each tower from the fire?

2. From the top of a cliff 180 m in height, the angles of depression to points B and C on the ground and in line with the base of the cliff are 50° and 20° respectively. Find the distance from B to C.

3. Two ships leave port A at the same time. One ship sails 300 km to port B in a direction 40° west of north. The other ship sails 500 km to port C in a direction 10° west of south. How far apart are ports B and C?

4. A person west of a tower notes that the angle of elevation of the top of the tower is 53°. A person east of the tower notes that the angle of elevation of the top of the tower is 37°. If the tower is 230 m in height, how far apart are the 2 people?

5. Two sides of a triangle are 35 cm and 43 cm, and the angle contained between them is 64°. What is the length of the third side?

6. From a point A the angle of elevation of the top of a cliff is 35°. From a point B, in a straight line between A and the base of the cliff C, the angle of elevation is 46°. If the distance from A to B is 120 m, how high is the cliff?

7. A bridge is 400 m in length. Observed from one end of the bridge, point A, a boat on the river beneath has an angle of depression of 62°. From the other end of the bridge, point B, the angle of depression of the same boat is 32°.

   a) What is the distance from point A on the bridge to the boat below?
   b) What is the perpendicular distance from a point on the bridge to the boat below?

8. The distance from a point A on the shore to boats B and C on the water is 520 m and 430 m, respectively. If the two boats are 880 m apart, calculate the size of ∠BAC.

9. A ladder, 10 m long, can just reach the top of a building when the ladder makes an angle of 70° with the ground. If the base of the ladder is moved another metre from the wall:

   a) Where will the top of the ladder be now, in relation to the top of the building?

   b) What angle will the ladder now make with the ground?

10. Find the smallest angle of the triangle with sides 8 cm, 10 cm and 12 cm. (Note: The smallest angle is always opposite the shortest side.)

Answer Key

1,2 1. a) $AB \leftrightarrow DE$ $\quad \angle A \leftrightarrow \angle D$
$BC \leftrightarrow EF$ $\quad \angle B \leftrightarrow \angle E$
$AC \leftrightarrow DF$ $\quad \angle C \leftrightarrow \angle F$

b) $XZ \leftrightarrow RT$ $\quad \angle Y \leftrightarrow \angle S$
$YZ \leftrightarrow RS$ $\quad \angle X \leftrightarrow \angle T$
$XY \leftrightarrow ST$ $\quad \angle Z \leftrightarrow \angle R$

c) $JK \leftrightarrow MN$ $\quad \angle H \leftrightarrow \angle P$
$HJ \leftrightarrow MP$ $\quad \angle K \leftrightarrow \angle N$
$HK \leftrightarrow NP$ $\quad \angle J \leftrightarrow \angle M$

d) $AD \leftrightarrow BD$ $\quad \angle C \leftrightarrow \angle E$
$BC \leftrightarrow DE$ $\quad \angle B \leftrightarrow \angle ADE$
$DC \leftrightarrow AE$ $\quad \angle BDC \leftrightarrow \angle A$

e) $XY \leftrightarrow ST$ $\quad \angle XZY \leftrightarrow \angle SZT$
$XZ \leftrightarrow TZ$ $\quad \angle X \leftrightarrow \angle T$
$YZ \leftrightarrow SZ$ $\quad \angle Y \leftrightarrow \angle S$

f) $BD \leftrightarrow DB$ $\quad \angle A \leftrightarrow \angle C$
$AB \leftrightarrow CD$ $\quad \angle ABD \leftrightarrow \angle BDC$
$AD \leftrightarrow BC$ $\quad \angle ADB \leftrightarrow \angle DBC$

g) $DE \leftrightarrow HK$ $\quad \angle DFE \leftrightarrow \angle HFK$
$DF \leftrightarrow HF$ $\quad \angle H \leftrightarrow \angle D$
$EF \leftrightarrow KF$ $\quad \angle K \leftrightarrow \angle E$

h) $MP \leftrightarrow PR$ $\quad \angle M \leftrightarrow \angle R$
$NP \leftrightarrow PS$ $\quad \angle N \leftrightarrow \angle S$
$MN \leftrightarrow RS$ $\quad \angle MPN \leftrightarrow \angle RPS$

3 1. a) $\frac{12}{x} = \frac{8}{16}$; $x = 24$ cm

$\frac{12}{20} = \frac{8}{y}$; $y = 13\frac{1}{3} = 133.3$ mm

b) Note: All sides of an equilateral triangle are of the same length. Therefore, $y = 4$ cm and $x = 5$ cm. Equivalent ratios are redundant for this exercise.

c) Note: In isosceles triangles, equal angles have equal sides opposite them. Therefore $x = 6.6$ cm and $y = 2.2$ cm. Equivalent ratios are redundant in this exercise.

d) $\frac{8}{x} = \frac{20}{15}$; x = 6 cm

$\frac{10}{8} = \frac{y}{20}$; y = 25 cm

e) x = 10 cm, y = 3.5 cm

f) $\frac{x + 2.5}{6} = \frac{2.5}{1.5}$; x = 7.5 cm

$\frac{y + 6}{6} = \frac{y}{1.5}$; y = 2 cm

g) x = 4.5 cm; y = 8 cm

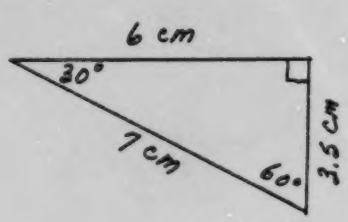
h) x = 14.4 cm; y = 25/7 cm

4

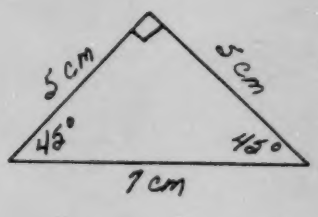
1.

a)	13	5	12	$5/13$	$12/13$	$5/12$
b)	$4\sqrt{2}$	4	4	$1/\sqrt{2}$	$1/\sqrt{2}$	1
c)	10	$\frac{6}{\sqrt{3}}$	8	$6/10$	$8/10$	$6/8$
d)	$\frac{2}{\sqrt{34}}$	5	1	$\sqrt{3}/2$	$1/2$	$\frac{\sqrt{3}}{3}$
e)	$\sqrt{34}$	5	3	$5/\sqrt{34}$	$3/\sqrt{34}$	$5/3$

2. a)



b)



Function/Angle	60	30	45
Sine	0.85714	0.50000	0.71429
Cosine	0.50000	0.85714	0.71429
Tangent	1.71429	0.58333	1.00000

5

1. a) 0.86603
b) 0.50000
c) 1.73205
d) 0.50000
e) 0.86603
f) 0.57735
g) 0.70711
h) 0.70711
i) 1.00000

2. a) 0.17365
   b) 0.92051
   c) 1.07237
   d) 0.98769
   e) 0.96126
   f) 1.66428
   g) 0.76604
   h) 0.30902
   i) 0.03492
   j) 0.89101
   k) 0.01745
   l) 0.60086

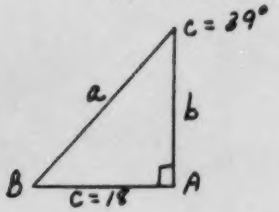
6

1. a) $\angle A = 45^\circ$; b = 10.61
   b) $\angle F = 30^\circ$; e = 14
   c) $\angle S = 70^\circ$

Sine 20° = 5/t
t = 5/sine 20
t = 14.62

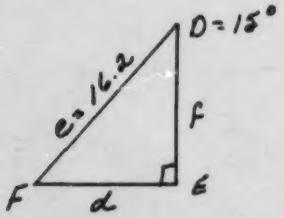
Tan 20° = 5/s
s = 5/tan 20
s = 13.74

2. a) $\angle A = 48^\circ$, c = 13.38, a = 14.86
   b) $\angle F = 35^\circ$, d = 6.102, e = 4.99
   c)



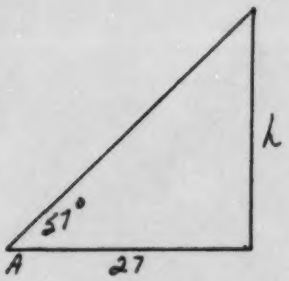
$\angle B = 51^\circ$
a = 28.6
b = 22.23

   d)



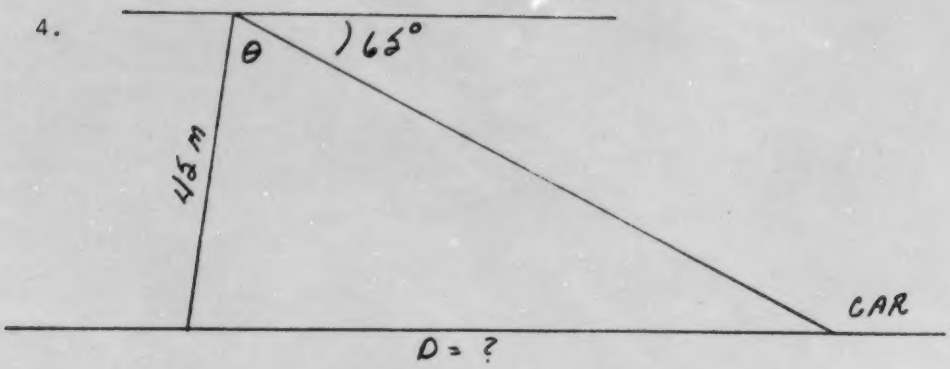
$\angle F = 75^\circ$
f = 15.65
d = 4.19

3.



Tan 57 = h/27
h = 27(Tan 57)
h = 41.58 m

4.

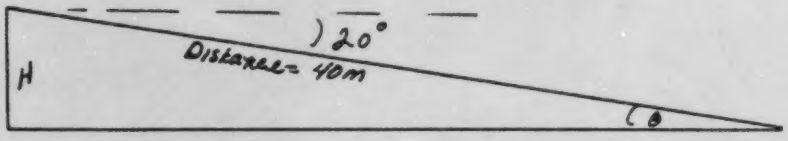


$\theta = 35^{\circ}$

Tan θ = D/45

D = 45(Tan 35); D = 31.51 m

5.



$\theta = 20^{\circ}$

Sine θ = H/40

H = 40(Sine 20) = 13.68 m

7

1. a) $\theta = 36.87^{\circ}$
   b) $\theta = 45^{\circ}$
   c) $\theta = 53.13^{\circ}$
   d) $\theta = 45^{\circ}$
   e) $\theta = 48.19^{\circ}$
   f) $\theta = 24.58^{\circ}$

2. a) $\theta = 15^{\circ}$
   b) $\theta = 72^{\circ}$
   c) $\theta = 55^{\circ}$
   d) $\theta = 36^{\circ}$
   e) $\theta = 28^{\circ}$
   f) $\theta = 45^{\circ}$
   g) $\theta = 61^{\circ}$
   h) $\theta = 3^{\circ}$
   i) $\theta = 88^{\circ}$

8 1. a) $x = 25$, $\theta = 73.74^\circ$, $\gamma = 16.26^\circ$ b) $x = 6.508$, $\theta = 56.54^\circ$, $\gamma = 33.46^\circ$ c) $x = 9$, $\theta = 77.32^\circ$, $\gamma = 12.68^\circ$

2. a) $c = 7.2$, $\angle C = 47.92^\circ$, $\angle B = 42.08^\circ$ b) $f = 17.2$, $\angle D = 54.46^\circ$, $\angle E = 35.54^\circ$ c) $t = 34.56$, $\angle T = 45.75^\circ$, $\angle R = 44.25^\circ$

3. 28.07° = angle of elevation

4. $\theta = 55.08^\circ$, $b = 5.72$ m

5. $AC = 9.7$, $\theta = 47.92^\circ$

9. 1. a) 0.98481
b) -0.5
c) -0.57735
d) 0.70711
e) -0.98481
f) -11.43005
g) 0.25882
h) -1.0
i) -1.96261
j) 1.0

10 1. a) $\angle A = 180 - (55 + 65) = 60^\circ$

$$\frac{b}{\text{Sine } B} = \frac{c}{\text{Sine } C} \text{ and } \frac{b}{\text{Sine } 55} = \frac{15}{\text{Sine } 65}$$

$$b = \frac{14(\text{Sine } 55)}{\text{Sine } 65} = 12.65$$

$a = 13.38$

b) $$\frac{e}{\text{Sine } E} = \frac{f}{\text{Sine } F} \text{ and } \frac{6.5}{\text{Sine } 125} = \frac{3}{\text{Sine } F}$$

$\angle F = 22.21^\circ$

$\angle D = 180 - (125 + 22.21) + 32.79^\circ$

$d = 4.3$

c) $t = 15$

$R = 53.13^\circ$

$T = 36.87^\circ$

2. a) $17.54 = z$
   b) $10.44 = n$
   c) $\angle H = 15.07^\circ$

11. 1. a) $AC = 3.78$
    b) $DF = 6.2$
    c) $RT = 10$
    d) $\angle H = 75^\circ$
    e) $\angle X = 21.6^\circ$
    f) $\angle N = 152^\circ$

12. 1. West tower is 20.5 km away
    East tower is 18.4 km away

    2. BC = 343.5 m

    3. B to C is 730 km

    4. They are 478.5 m apart

    5. Third side is 41.89 cm

    6. 259.5 m

    7. a) 212.5 m
       b) 187.6 m

    8. $\angle A = 135.5^\circ$

    9. a) 0.427 lower
       b) $\theta = 63.77^\circ$

    10. 41.41°

C.IV. STATISTICS (1 module)

1

1. Explain briefly how statistics are used by teachers, administrators, etc., in schools.

2. Explain briefly how statistics are used by governments. For what purposes are they used?

3. How would information (statistics) on population distributions be of value to an automobile manufacturer?

4. Why would manufacturers keep statistics on products they manufacturer?

5. Briefly discuss the effect that statistics and their use have on your everyday life.

2,3

For each of the following, draw a bar graph and estimate the value of the central tendency from the bar graph. NOTE: All students shouls use same data.

1. the weights of 7 members of your class
2. the heights of 9 members of your class
3. The time (in minutes) taken for 8 of your classmates to get to school in the morning
4. 47, 47, 42, 44, 45, 55
5. 62, 68, 69, 70, 68, 58, 69
6. 125, 150, 96, 112, 130, 127, 105
7. 36, 40, 40, 39, 47, 37, 38, 35
8. 68°, 58°, 76°, 74°, 69°, 65°
9. 84%, 95%, 83%, 65%, 66%, 86%, 79%
10. 215, 267, 250, 247, 290, 258, 232, 186

4

1. Define each of the following:
   a) mean (arithmetic)
   b) median
   c) mean deviation

2. Why would clothing manufacturers be interested in arithmetic means?

3. Give a situation in which the arithmetic mean is not a good measure of central tendency.

4. Give a situation in which the median is not a good measure of central tendency.

5. Under perfect conditions, what would be the relationship between the mean and the median?

6. Calculate the mean and mean deviation for each of the following:

a) $30 000; $34 000; $36 000; $38 000; $86 000; $74 000
b) 20, 17, 23, 8, 4, 5, 3, 1, 14
c) 65, 38, 86, 95, 46, 72, 85
d) 40, 52, 48, 35, 30, 29, 33, 28
e) 361, 60, 344, 472, 3335, 2010, 369, 353, 531, 487
f) 6.0, 5.8, 4.9, 4.8, 6.2, 6.4, 6.6, 7.0, 6.7, 6.4, 8.1, 8.4, 5.4
g) 12 salaries of $13 500; 8 salaries of $17 000; and 4 salaries of $21 300
h) 6 tractors at 6040 kg, 4 tractors at 8280 kg, and 2 at 12 040 kg
i) 30 people 183 cm tall, 25 at 175 cm tall, 15 at 170 cm tall, 8 at 167 cm, and 3 at 160 cm
j) 8 marks of 93%, 6 of 87%, 5 of 76%, 12 of 69%, 3 of 60%, 2 of 43%
k) 1 pig weighing 136 kg, 3 at 144 kg, 7 at 129 kg, and 5 at 154 kg

7. Find the median of each of the sets of data in questions 6 (a-f).

5 Find the arithmetic mean and median for each of the following sets of grouped data.

1.

Weight	$\bar{x}$	$f(\bar{x})$
88 - 92	90	8
83 - 87	85	14
78 - 82	80	51
73 - 77	75	17
68 - 72	70	10

Weights of 100 people in kg

2.

Points	$\bar{x}$	$f(\bar{x})$
117-123	120	6
110-116	113	16
103-109	106	23
96-102	99	19
89- 95	92	12
82- 88	85	4

Points scored by an N.B.A. team in 80 games

3.

Bushels/Acre	$\bar{x}$	$f(\bar{x})$
34 - 37	35.5	7
30 - 33	31.5	6
26 - 29	27.5	7
22 - 25	23.5	27
18 - 21	19.5	32
14 - 17	15.5	13
10 - 13	11.5	8

Yield/acre of 100 wheat fields surveyed

4.

Test Scores	$\bar{x}$	$f(\bar{x})$
88 - 100	94	8
75 - 87	81	14
62 - 74	68	45
49 - 61	55	10
36 - 48	42	6

Scores of 83 students writing a 301 Mathematics test

5.

Weights	$\bar{x}$	$f(\bar{x})$
500-525	512.5	4
474-499	486.5	10
448-473	460.5	18
422-447	434.5	22
396-421	408.5	6
370-395	382.5	5

Weights (kg) of 65 steers brought to market.

6

1. a) Define the terms 'sample' and 'reliability'.
   b) Why do statisticians use samples?
   c) Describe some precautions one must take in selecting samples.

2. Using the following data, find the information requested.
   a) find the mean using first two numbers
   b) find the mean using first four numbers
   c) find the mean using first ten numbers
   d) find the mean using all the data
   e) discuss the results of the above

   6, 7, 10, 6, 8, 14, 10, 14, 5, 8, 9, 8, 10, 8, 7, 13, 17, 14, 5, 8, 9, 7, 8, 8, 7, 15, 15, 15, 5, 8

3. Using the following data, repeat the five steps of problem 2 above.

   90, 72, 5, 6, 7, 106, 33, 9, 3, 9, 52, 244, 97, 4, 143, 139

4. Using the following data, repeat the five steps of problem 2 above.

   46, 4, 22, 34, 314, 239, 25, 57, 26, 81, 12, 60, 1

7.

1. For second and fourth problems in section 5, calculate the following:
   a) first quartile mark
   b) second quartile mark
   c) third quartile mark
   d) second quintile mark
   e) fourth quintile mark
   f) 62nd percentile mark
   g) 43rd percentile mark
   h) 7th decile mark

8

1. Using the information in problems 1 and 3 of section 5, construct a cumulative frequency graph. Find the 30th and 65th centiles from the graph.

Review

The sets of data provided at the end of the notes of this module can be used in whole or in part for the purposes of review.

Answer Key

4

6.

	Mean	Mean Deviation
a)	$49 666.67	$20 222.22
b)	10.56	7.06
c)	69.57	17.06
d)	36.88	7.35
e)	832.2	736.12
f)	6.36	.78
g)	$15 966.67	$2466.67
h)	7786.67 kg	1417.78 kg
i)	169.77 cm	7.19 cm
j)	76.11%	11.16%
k)	140.06 kg	10.19 kg

7. a) $36 000
b) 14
c) 72
d) 34
e) 420.5
f) 6.4

5

	Mean	Median
1.	79.65 kg	79.75 kg
2.	103.64 pts	104.02 pts
3.	21.82 bushels	21.13 bushels
4.	69.25	69.01
5.	448.1 kg	447.50 kg

6

2. a) 6.5
b) 7.25
c) 8.80
d) 9.37

3. a) 81
b) 43.25
c) 34
d) 63.69

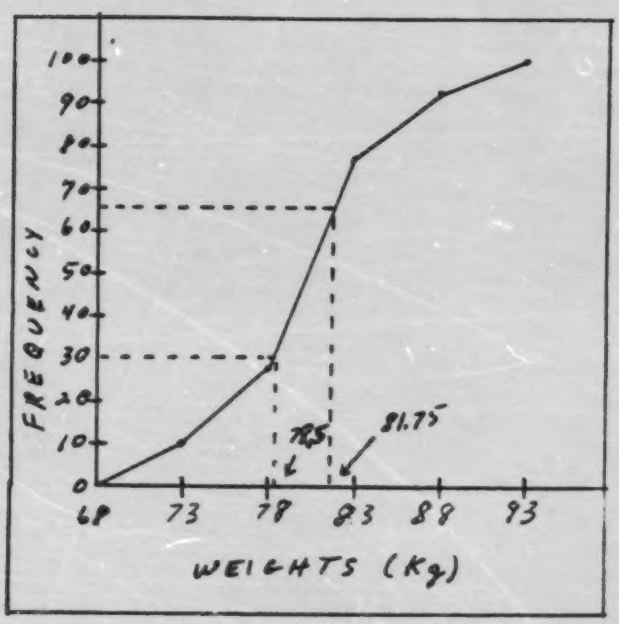
4. a) 25
b) 26.5
c) 84.8
d) 70.85

7

	2nd problem	4th problem
1st quartile	96.97	62.94
2nd quartile	104.02	69.01
3rd quartile	110.38	75.43
2nd quintile	101.39	65.83
4th quintile	112.13	79.14
62nd percentile	107.07	71.61
43rd percentile	102.13	67.28
7th decile	108.89	73.63

NOTE: In calculating the above answers, the terms used were the nearest whole number. For example, in calculating the 62nd percentile of problem 2, the term used was the 50th term (i.e., .62 x 80 = 49.6 or 50).

8 1.



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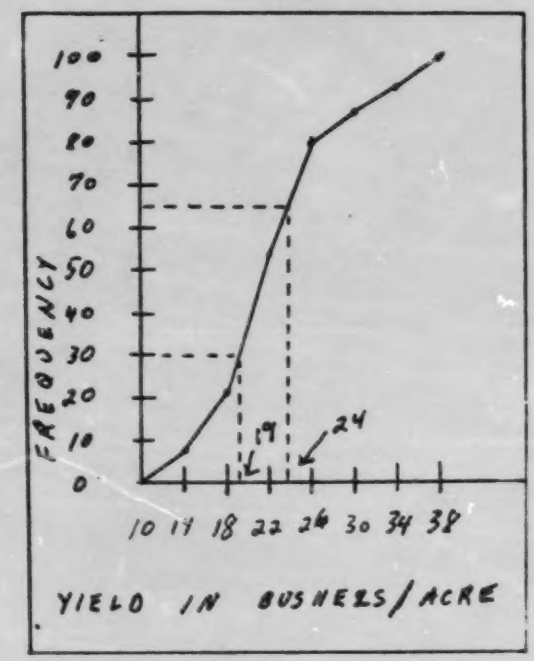

100
90
80
70
60
50
40
30
20
10
0
FREQUENCY
19
24
10 14 18 22 24 30 34 38
YIELD IN BUSHELS/ACRE

OP.1. ANNUITIES

1

1. Compute the compound amount, after 5 years, if $4000 has been invested at 9% and compounded quarterly.

2. Bert Lougheed established an education fund for his daughter by investing $6000 at 10% in 1980. If the interest was compounded semi-annually:

   a) What would the total amount be after 15 years?
   b) How much of the amount would be due to accrued interest?
   c) Express the increased buying power of the $6000, after 15 years, as a percent rate, if inflation over the 15 year period was 180%.

2

1. Eugene Picklyk wanted to invest enough money at 14% compounded quarterly so that in eight years his son would have $7500 to spend on an automobile. How much should he invest. (Calculate the present value.)

2. The compound amound of an invested principal had grown to $22 500.00 in 16 years. The money earned interest at the rate of 12% compounded annually. What was the original principal?

3,4

1. Match the following:

Types		Description
1. Ordinary	____	a. Begins on a definite date but the ending date depends on a future event.
2. Contingent	____	b. These begin on a definite date but the length of the term is undefined.
3. Deferred	____	c. Begin and end on pre-arranged dates.
4. Annuity Certain	____	d. The payments are made at the end of each interval but the first interval does not begin until some later time.
5. Perpetuity	____	e. Payments are made at the end of each interval.
6. Annuity Due	____	f. Payments are made at the beginning of each interval.

5 Indicate whether the following statements are true or false.

1. In a complex annuity the payment interval and the interest conversion are identical.

2. An annuity is classified as an ordinary annuity because of the date of the payments.

3. An ordinary simple annuity is one in which the payments are made at the end of the interval.

4. There is no such thing as an ordinary complex annuity.

5. A deferred annuity is also automatically a perpetual annuity.

6. The termination date of a contingent annuity depends on some future event.

7. The word 'annuity' implies a series of periodic payments.

6 1. Mr. Waddell decided to invest all the Family Allowance cheques for his son until his fifteenth birthday. He was able to arrange a scheme whereby the investment would bear 12% for 15 years. What amount of money would be accumulated by this plan if the monthly allowance was $18?

2. Calculate the regular payment required to accumulate $3800.00 over a 5 year period if the money bears 15% compounded quarterly. This is a simple annuity; i.e., the payment interval and the interest conversion period are the same.

7 1. An ordinary simple annuity calls for $400 to be paid every 6 months bearing interest at 10% for 8 years, compounded semiannually. Calculate the present value.

2. What is the present value of an ordinary simple annuity by which the monthly family allowance payments of $18 are invested for 15 years at 12% compounded monthly?

8 1. In a complex annuity, the payment interval and the conversion period are not the same. For example, if you put away $30 every month for 5 years and your money bears 12% interest compounded semiannually, you have a complex annuity. Calculate the amount of this annuity.

2. If the amount, Sn, of a complex annuity is $6500, calculate the regular payment required if the payment is to be made monthly, bearing 15% compounded quarterly, for 7 years.

9 1. Compute the present value of the annuity described in Objective 8, Question 1.

2. Compute the present value of the annuity described in Objective 8, Question 2.

Formulae

Compound amount

$$A = P(1 + r)^n$$

Present value or principal of a compound amount

$$P = \frac{A}{(1 + r)^n}$$

Amount of an ordinary simple annuity

$$S_n = R \frac{(1 + i)^n - 1}{i}$$

Present value of an ordinary simple annuity

$$A_n = R \frac{1 - (1 + i)^{-n}}{i}$$

Amount of an ordinary complex annuity

$$S_n = R \frac{(1 + i)^{nc} - 1}{(1 + i)^c - 1}$$

Present value of an ordinary complex annuity

$$A_n = R \frac{1 - (1 + i)^{-nc}}{(1 + i)^c - 1}$$

Mortgage balance after n payments

$$B_n = P(1 + r_m)^n - A \frac{(1 + r_m)^n - 1}{r_m}$$

Answer Key

1

1. $A = P(1 + r)^n = \$6242.04$

2. a) A = \$12 473.57
   b) Accrued interest is \$6473.57
   c) 15.5%

2

1. $P = \frac{A}{(1 + r)^n}$; P = \$2494.42

2. P = \$18 590.64

3,4

1. e
2. a
3. d
4. c
5. b
6. f

5

1. F
2. T
3. T
4. F
5. F
6. T
7. T

6

1. $S_n = \$8992.44$

2. R = \$130.96

7

1. \$4335.11
2. \$1499.79

8

1. \$2431.19
2. \$44.51

9

1. \$1357.56
2. \$2318.97

OP.11. DEPRECIATION AND DISTRIBUTION OF OVERHEAD COSTS (1 module)

A Depreciation

A-1 No exercises required for definition.

A-2
1. Robertson's Mfg. Co. purchased an office desk for $240. Ten years later, it was sold for $35. What was the total depreciation on the desk?

2. A student purchased a typewriter for $160 when he started Grade X. After graduation from Grade XII, 3 years later, he traded it on an electric machine. Its trade-in value was $50. What was the total depreciation on the typewriter?

3. A young couple purchased a freezer for $329.99, used it for 15 years. At the end of that time the freezer was completely worn out, and had to be scrapped. Find the total depreciation on the freezer.

A-3
1. Delta Delivery Co. purchased a van for $13 500 in 1980. Three years later it is sold for $5500. Using the straight-line method of depreciation, calculate a) total depreciation, b) annual depreciation, c) monthly depreciation.

2. Aldershaw Appliance Repairs purchased tools and equipment to a value of $27 000 when the company first started in business. Eight years later, they replaced it, and were given a trade-in allowance of $3000. Find a) total depreciation, b) annual depreciation, c) monthly depreciation.

3. The Aldershaw company purchased a copying machine for their establishment at a total cost of $2500. After 15 years, the copier was worn out and had to be replaced. Calculate a) total depreciation, b) annual depreciation, c) monthly depreciation.

4. Aldershaw's also bought six vans when they started in business, at a cost of $12 500 each. When they were traded in four years later, they were valued at $4200 each. For the entire fleet of vans, find a) total depreciation, b) annual depreciation, c) monthly depreciation.

A-4 No exercises.

A-5

1. A truck is purchased for $38 800. If it depreciates by $7500 annually, find the book value at the end of each of the first three years.

2. An automobile is purchased for $12 400. Four years later, it is valued at $4600. a) Find the total depreciation. b) Find the annual depreciation. c) Find the book value of the automobile at the end of each of the four years.

3. Sandy's Soupery keep kitchen appliances at a total cost of $2510. If the appliances are worthless at the end of ten years, calculate a) the amount of annual depreciation, and b) the book value of the appliances at the end of the seventh year.

A-6

1. Complete the following table:

No.	Item	Price	Disposal Value	Years In Use	Annual Deprec'n	Book Value 3rd Year	% Rate of Deprec'n
1	Calculator	265.95	50.00	8			
2	Typewriter	844.00	150.00	6			
3	Copier	8400.00	2500.00	15			
4	Desk	585.00	212.00	12			
5	Filing Cabinet	362.00	75.00	12			
6	Chair	128.50	15.00	10			

2. You purchase a stereo for $895.00. After 12 years, it has a trade-in value of $130. Find a) total depreciation, b) annual depreciation, c) annual % rate of depreciation.

3. A wristwatch is priced at $260. It is worn for 12 years and then is no longer useful. Find the annual % rate of depreciation.

4. Rob James buys a camper for $16 800. Five years later, he sells it to a neighbour for $6200. Calculate the annual % rate of depreciation.

5. A classroom set of textbooks costs $426. If the books are completely worn out after 9 years, find the annual % rate of depreciation.

6. Lab equipment is purchased for a new clinic, and used for six years before being traded in on newer models. If the initial cost was $263 527.83, and the trade-in allowance amounted to $97 622.67, find the annual rate of depreciation.

7. A householder pays $1100 to have the roof of his house shingled. Twenty years later, the shingles must be replaced. Calculate the annual % rate of depreciation.

A-7 1. Complete the following table:

No.	Asset	Cost	Disposal Value	Time	Total Deprec'n	Annual Deprec'n	Annual % Deprec'n Straight Line	Annual % Deprec'n Declining Balance
1	Rug	2400	550	20 years				
2	Sofa	1395	345	10 years				
3	China Cabinet	1200	300	15 years				
4	Bedroom Suite	2150	525	12 years				
5	Stove	775	125	8 years				

2. Complete the following table:

Year	Bond Value	% Rate of Deprec'n	Amount of Deprec'n
1	$12 000	22%	
2		22%	
3		22%	
4		22%	
5		22%	

3. Nora Lister buys a motorbike for $2000. Calculate the book value of her bike after 3 years, if the declining balance rate of depreciation is 20%.

4. A dining room suite is priced at $1500. If the rate of depreciation on the declining balance is 18%, find the book value at the end of the fourth year of ownership.

5. 'The Mice,' a rock group, purchased sound equipment at a cost of $3600. Calculate the book value of the equipment after 2 years, if the declining-balance method is used, at an annual rate of 25%.

6. The Clayville Fire Department purchases a new pumper unit at a total cost of $96 200.00. They compute depreciation by the declining-balance method, at an annual rate of 12%. Find the total depreciation and book value at the end of each of the first 3 years.

A-8 1. Using the sum of the years digits method, calculate annual depreciation for each of the items listed, for years 1, 2 and 3 as indicated.

Item	Price	Scrap Value	Years Held	Year 1	Year 2	Year 3
1	2650	850	6			
2	3400	1600	4			
3	1680	460	8			
4	1200	325	7			
5	990	140	5			
6	800	175	3			

2. Boswell Ames purchased a snowblower for $560. At the end of 4 years, he traded it on a new machine and received a $200 trade-in allowance. Calculate the depreciation for each of the four years, by the sum of the digits method.

3. Harley Wintergreen bought a grand piano for $12 000, kept it for 18 years and then sold it for $7200. Calculate the depreciation for each of the first two years, using the sum of the digits method.

A-9 1. The Dandy Distributorship purchased a new delivery van at a price of $12 000. The company expects to trade in the van after 5 years for a value of $7000. Calculate the depreciation and the book value for each of the 5 years a) by the straight line method, b) by the declining balance method, and c) by the sum of the digits method.

B Distributing Overhead Costs

B-1 1. The Rombough Manufacturing Co. pays an annual rent of $36 000 for its business premises. Calculate the amount to be charged to each department if costs are allocated according to floor space occupied.

Department	Floor Space Occupied	Share of Rent
Sales	160 m^2	
Production	2000 m^2	
Packaging	840 m^2	
Clerical	120 m^2	
Administration	80 m^2	

2. A community recreation complex allocates utilities expense according to floor space occupied. Calculate the amount to be charged to each department if the total cost of utilities amounts to $1864.00 per month.

Department	Floor Space Occupied	Share of Utilities Expense
Skating Rink	1500 m^2	
Swimming Pool	600 m^2	
Bowling Alley	400 m^2	
Crafts Rooms	100 m^2	
General Purpose Hall	480 m^2	
Exercise Rooms	64 m^2	
Games Room	36 m^2	

B-2 1. Anderson's Real Estate has a monthly overhead expense of $1240.00. Allocate the cost to each department according to the number of employees in each.

Department	No. of Employees	Overhead Expense
Administration	8	
Property Management	7	
Advertising	3	
Clerical	21	
Mortgage	15	
Sales	60	

2. Morrow's Department Store has annual overhead of $36 420. Calculate the amount to be allocated to each of the following departments, on the basis of the number of employees in each.

Department	No. of Employees	Overhead Expense
Shoes	6	
Clothing	8	
Furniture	2	
Hardware	3	
Appliances	1	
Linens/Bedding	4	
Entertainment	5	
Clerical	12	
Books & Stationery	7	
Administration	9	

B-3 1. Gordon's Stereo Supply Company allocates expenses based on the sales volume of each department. Total overhead expenses in August amounted to $1620.00. Calculate the amount to be charged to each department.

Department	Sales Volume	Overhead Expense
A	$2200	
B	2545	
C	2545	
D	6390	
E	5535	

2. Carol's Cosmetics Supply Company has overhead expense amounting to $580 for the month of September. How much of this expense will be allocated to each department, on the basis of sales volume?

Department	Sales Volume	Overhead Expense
A	$1590	
B	342	
C	2505	
D	752	
E	826	

Answer Key

A-1 No exercises

A-2
1. 240 - 35 = $205
2. 160 - 50 = $110
3. $329.99

A-3
1. 13 500 - 4500 = $9000 total depreciation
   9000 ÷ 3 = 3000 annual depreciation
   3000 ÷ 12 = 250 monthly depreciation

2. a) $24 000
   b) $3000
   c) $250

3. a) $2500
   b) $166.67
   c) $13.89

4. a) $8300
   b) $2075
   c) $172.92

A-4 No exercises.

A-5
1. $38 800 - 7500 = $31 300 after 1 year
   $31 300 - 7500 = $23 800 after 2 years
   $23 800 - 7500 = $16 300 after 3 years

2. a) $7800.00
   b) $1950.00
   c) $12 400 - 1950 = 10 450 book value after 1 year
      $10 450 - 1950 = 8 500 book value after 2 years
      $ 8 500 - 1950 = 6 550 book value after 3 years
      $ 6 550 - 1950 = 4 600 book value after 4 years

3. a) $2510
   b) $2510 ÷ 10 = 251 depreciation per year
      251 x 7 = 1757 total depreciation after 7 years
      2510 - 1757 = $753 book value after 7 years

1.

A-6

1	$ 26.99	$184.98	10.1%
2	115.67	496.99	13.7
3	393.33	7220.01	4.7
4	31.08	491.76	5.3
5	23.92	290.24	6.6
6	11.35	94.45	8.8

2. a) 895 - 130 = $765
   b) 765 ÷ 12 = 63.75
   c) 63.75/895 x 100 = 7.1%

3. 8.3%
4. 12.6%
5. 11.1%
6. 10.5%
7. 5%

A-7

1.

1	1850	92.50	3.9%	7.8%
2	1050	105.00	7.5%	15%
3	900	60.00	5.0%	10%
4	1625	135.42	6.3%	12.6%
5	650	81.25	10.5%	21%

2.

1	$12 000.00	22%	$2640.00
2	9 360.00	22%	2059.20
3	7 300.80	22%	1606.18
4	5 694.62	22%	1252.82
5	4 441.80	22%	977.20

3. $1024
4. $678.18
5. $2025

6.

Year 1:	$11 544.00	$84 656.00
Year 2:	$10 158.72	$74 497.28
Year 3:	$ 8 939.67	$65 557.61

A-8

1. 1 + 2 + 3 + 4 + 5 + 6 = 21; 2650 - 850 = 1800 total depreciation
   6/21 x 1800 = $514.29 in year 1
   5/21 x 1800 = $428.57 in year 2
   4/21 x 1800 = $342.86 in year 3)

1	$514.29	$428.57	$342.86
2	720.00	540.00	360.00
3	271.11	237.22	203.33
4	218.75	187.50	156.25
5	283.33	226.67	170.00
6	312.50	208.33	104.17

2. Year 1 $144
   2 108
   3 72
   4 36

3. Year 1 $505.26
   2 477.19

A-9 1. a) $12 000 - 7000 = 5000 total depreciation in 5 years
$5000 ÷ 5 = $1000 per year
12 000 - 1000 = $11 000 book value at end of Year 1
11 000 - 1000 = $10 000 book value at end of Year 2
10 000 - 1000 = $ 9 000 book value at end of Year 3
9 000 - 1000 = $ 8 000 book value at end of Year 4
8 000 - 1000 = $ 7 000 book value at end of Year 5

b) Rate is 21.5%

	Deprec'n	Book Value
Year 1	21.5% x 12 000 = 2580.00	12 000.00 - 2580.00 = 9420.00
Year 2	21.5% x 9 420 = 2025.30	9 420.00 - 2025.30 = 7394.70
Year 3	21.5% x 7394.20= 1589.86	9 394.70 - 1589.86 = 5804.84
Year 4	21.5% x 5804.84= 1248.04	5 804.84 - 1248.04 = 4556.80
Year 5	21.5% x 4556.80= 979.71	4 556.80 - 979.71 = 3477.09

c) 1 + 2 + 3 + 4 + 5 = 15

		Deprec'n	Book Value
Year 1	5/15 x 5000 =	$1666.67	12 000.00 - 1666.67 = 10 333.33
Year 2	4/15 x 5000 =	1333.33	10 333.33 - 1333.33 = 9 000.00
Year 3	3/15 x 5000 =	1000.00	9 000.00 - 1000.00 = 8 000.00
Year 4	2/15 x 5000 =	666.67	8 000.00 - 666.67 = 7 333.33
Year 5	1/15 x 5000 =	333.33	7 333.33 - 333.33 = 7 000.00

B-1 1. $1800; $22 500; $9450; $1350; $900
2. $879.25; $351.70; $234.47; $58.62; $281.36; $37.51; $21.10

B-2 1. $87.02; $76.14; $32.63; $228.42; $163.16; $652.63
2. $3833.68; $5111.58; $1277.89; $1916.84; $638.95; $2555.79; $3194.74; $7667.37; $4472.63; $5750.53

B-3 1. $171.18; $322.91; $198.03; $497.20; $430.68;
2. $153.32; $32.98; $241.55; $72.51; $79.65

OP.III. DIGITAL LOGIC

Review of the Binary Number System

1. Write the decimal number 3569 as the sum of products. Multiply each digit by the power of 10 it represents and add the results.

2. List the first 10 powers of 2. For example $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, etc.

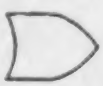
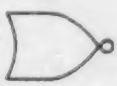
3. Let 101 101 be a number in the binary system. Each digit multiplies different power of 2. Evaluate the number as a sum of products and write the decimal equivalent number.

4. Express 75_{10} in binary code. The subscript 10 is to remind you that 75 is written in the decimal code (base 10).

5. Make a list of the numbers 0 to 15_{10} and give the binary code for each one.

6. Assume there are three connections on an electrical device. You are required to apply a 0 or a 1 to each connection. How many different ways can you apply a 0 and 1 to three connections? Make a list of all the possibilities.

III-1 1. Write the name of the gate that each symbol represents:

a) b)  c) d) 

III-2 1. How many different connection combinations of 1 and 0 can be applied to a 4-input gate.

III-3 1. The function of an AND gate is the same as 2 or more switches connected in _____.

2. The function of an OR gate is the same as 2 or more switches connected in _____.

3. The function of a NOR gate is the same as 2 or more switches connected in _____ followed by an _____.

III-4 1. Give the names and symbols of two gates which are controlled by a 1 (high) input.

2. Give the names and symbols of two gates which are controlled by a 0 (low) input.

III-5 1. Draw a two-input AND gate and prepare a truth table listing all the possible input combinations and the outputs.

2. Draw a two-input NOR gate and prepare a truth table listing all the possible input combinations and the outputs.

III-6 1. Draw the symbol for a three-input OR gate with one of the inputs inverted.

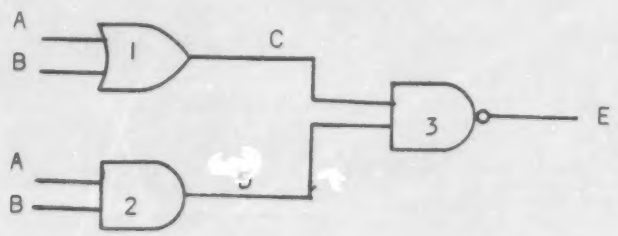
2. Draw the symbol for a four-input NAND gate with any two of the inputs inverted.

III-7 1. Give the decimal equivalent of the following binary numbers:

a) 101
b) 1101
c) 11
d) 00101
e) 10100
f) 100
g) 1111

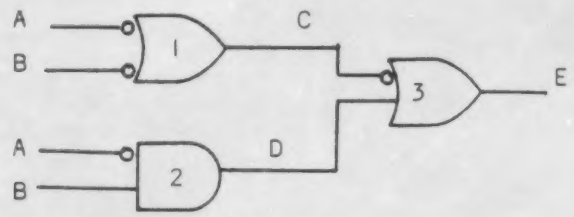
III-8 1. Prepare a truth table for a 4-input AND gate. Complete the output column.

III-9 1. Prepare a truth table showing all the inputs and outputs in the following combination of gates:

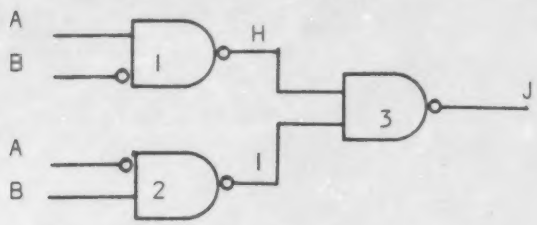


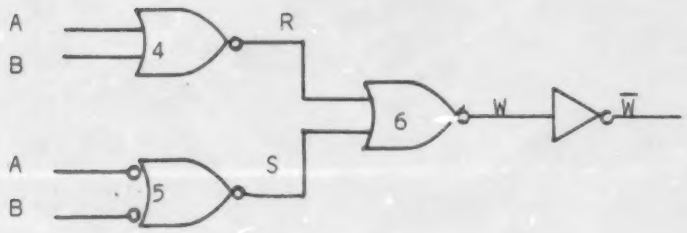
III-10 1. What is meant by inverting the logic level of a signal?

III-11 1. Prepare a truth table showing the inputs and outputs in the following combination of gates:



III-12 1. Prepare a truth table to compare the outputs of these two circuits, i.e., compare J and W and $\overline{W}$. ($\overline{W}$ is read as not W)





III-13 1. Explain the main difference between combinational and sequential logic.

III-14 1. Draw the symbol for a latch or flip-flop and state its most significant function.

III-15 1. Draw the schematic of the internal circuit of a memory cell.

2. Assume the initial condition of a latch is SET. List and explain the output of the latch as high and low (1 and 0) signals are applied to the SET and CLEAR inputs. The inputs are controlled by spring loaded switches.

III-16 1. Show the output wave of a simple set/re-set latch given the input signal waves.

III-17 1. Explain the following:
a) MSB
b) LSB
c) cascaded latches

III-18 1. Assume 10 pulses are applied to the switch of the binary counter. Show the output on the LEDs after every pulse.

Answer Key

Review of Binary Code

1. $3 \times 10^3 + 5 \times 10^2 + 6 \times 10^1 + 9 \times 10^0$
   The digits from left to right have weighted values since they represent descending powers of 10.

2. $2^0 = 1$ $\quad 2^4 = 16$ $\quad 2^8 = 256$
   $2^1 = 2$ $\quad 2^5 = 32$ $\quad 2^9 = 512$
   $2^2 = 4$ $\quad 2^6 = 64$
   $2^3 = 8$ $\quad 2^7 = 128$

   Note: $2^0 = 1$ because of the law of exponents which states that $a^x \div a^y = a^{x-y}$. Consider a numerical example:

   $5^3 \div 5^3 = \frac{5^3}{5^3} = 1$ and $5^3 \div 5^3 = 5^{3-3} = 5^0$

   Therefore $5^0 = 1$ and for any base, x; $x^0 = 1$

   This extraordinary property of the zero exponent guarantees there will be a unit (one) in any number system.

3. $1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 32 + 0 + 8 + 4 + 0 + 1 = 45_{10}$

4. 75 - 2 = 37 remainder 1 LSB
   37 - 2 = 18 remainder 1
   18 - 2 = 9 remainder 0
   9 - 2 = 4 remainder 1
   4 - 2 = 2 remainder 0
   2 - 2 = 1 remainder 0
   *1 - 2 = 0 remainder 1 MSB

   Therefore $75_{10} = 1001011_2$

   You should check this result the way question #3 is done.

   *Read each step in the above series of divisions with remainders very carefully - especially the last step.

5. | | | | |
   |---|---|---|---|
   | 0 | 0000 | 8 | 1000 |
   | 1 | 0001 | 9 | 1001 |
   | 2 | 0010 | 10 | 1010 |
   | 3 | 0011 | 11 | 1011 |
   | 4 | 0100 | 12 | 1100 |
   | 5 | 0101 | 13 | 1101 |
   | 6 | 0110 | 14 | 1110 |
   | 7 | 0111 | 15 | 1111 |

6. There will be 8 combinations since $2^3 = 8$. The most orderly and direct way is to write the binary number codes for the decimal numbers 0 to 7.

000	100
001	101
010	110
011	111

III-1 1. a) AND b) OR c) NAND d) NOR

III-2 1. $2^4 = 16$. Make a list of the binary code for decimal numbers from 0 to 15.

III-3 1. Series. If any switch in a series is open, the circuit is open. That is why we say an AND gate is controlled by a low signal (zero). Also, AND gates are inhibited by low inputs.

2. parallel. If three switches are connected in parallel, any one of them can activate the circuit. That is why we say an OR gate is controlled by a high signal (one). Also, OR gates are enabled by high inputs.

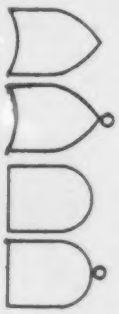
3. parallel, inverter

III-4 1. OR gate

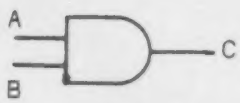
NOR gate

2. AND gate

NAND gate

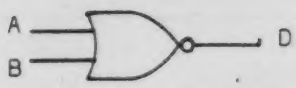


III-5 1.



A	B	C
0	0	0
0	1	0
1	0	0
1	1	1

Notice the input columns A and B simply reflect the binary coding of the decimal numbers 0 to 3; i.e., the first four binary numbers. Column C shows the output. Since a low signal controls the AND gate, it yields a low output whenever any one or both inputs are low.

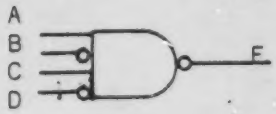
2. 

A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

Any high input to an OR gate will produce a high output. The NOR gate produces the complement because of the inverter at the output. NOTE: the complement of 0 is 1 and the complement of 1 is 0.

III-6 1.

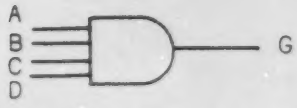
The symbol is correct if any one of the inputs is inverted.

2. 

The symbol is correct if any two of the inputs are inverted.

III-7 1. a) 5
b) 13
c) 3
d) 5; the leading zeros are not significant
e) 20
f) 4
g) 15

III-8



A	B	C	D	G
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

Since a low signal controls the AND gate the output is low if any one or all the inputs are low. The output is high only when all the inputs are high.

III-9 1.

A	B	C	D	E
0	0	0	0	1
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

III-10 1. Inverting the logic level means producing the complement. Zero (low) becomes high. One (high) becomes low.

III-11 1.

A	B	C	D	E
0	0	1	0	0
0	1	1	1	1
1	0	1	0	0
1	1	0	0	1

Since gate 1 is an OR gate, any high input produces a high output. Since both inputs are inverted, any low signal produces a high output (see column C). Gate 2 is an AND gate and any low input produces a low output (column D). Gate 3 is an OR gate with one input inverted.
High D produces high E.
Low C produces high E.
Others produce low E.

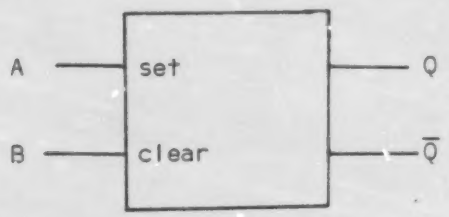
III-12 1.

A	B	H	I	J	R	S	W	$\overline{W}$
0	0	1	1	0	1	0	0	1
0	1	1	0	1	0	0	1	0
1	0	0	1	1	0	0	1	0
1	1	1	1	0	0	1	0	1
				*			*	*

Conclusion: Columns J and W are the same. The inverter after W produces the complement of W. ($\overline{W}$ is read not W)

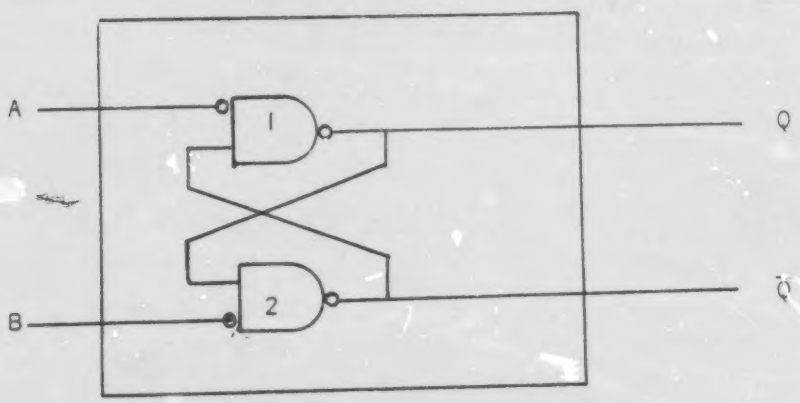
III-13 1. In a combinational circuit, the output is affected by every change in the input. In sequential logic a change in the input may not change the output. The sequence of changes in input and the nature of the devices, are important in determining the output. Sequential circuits will always involve at least one latch or memory cell.

III-14 1.



The set and clear conditions occur when A and B (respectively) have high signals.
Set condition $Q = 1$; $\overline{Q} = 0$
Clear condition $Q = 0$; $\overline{Q} = 1$
The output Q and $\overline{Q}$ always reflect the result of the last high pulse on A and B. Therefore the latch output always retains the result of the last high input signal; hence a memory cell.

III-15 1.



2. Pencil in a zero on $\overline{Q}$, A, B and one on Q. Study the effects of applying ones to A and B alternately.

	A	B	Q	$\overline{Q}$	Comments
a)	0	0	1	0	Set - stable
b)	1	0	1	0	remains set
c)	0	0	1	0	remains set
d)	0	1	0	1	clear - stable
e)	0	0	0	1	remains clear
f)	1	1	X	$\overline{X}$	ambiguous

a) In the initial condition $\overline{Q}$ supplies a low signal to gate 1 so this gate is inhibited and any change on A input will have no effect on the latch output.

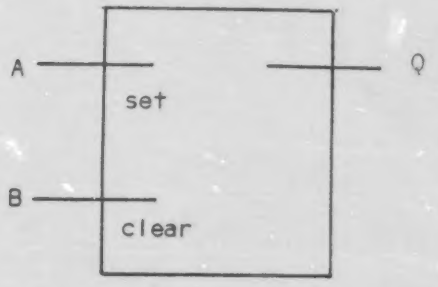
b),c) Refer to comment 'a' and trace the effect of the signals on A through the circuit.

d) The high signal on B does produce a change - the latch clears.

e) Changing B from high to low - will not change the output. (The latch remembers - return the effect of the last high signal.)

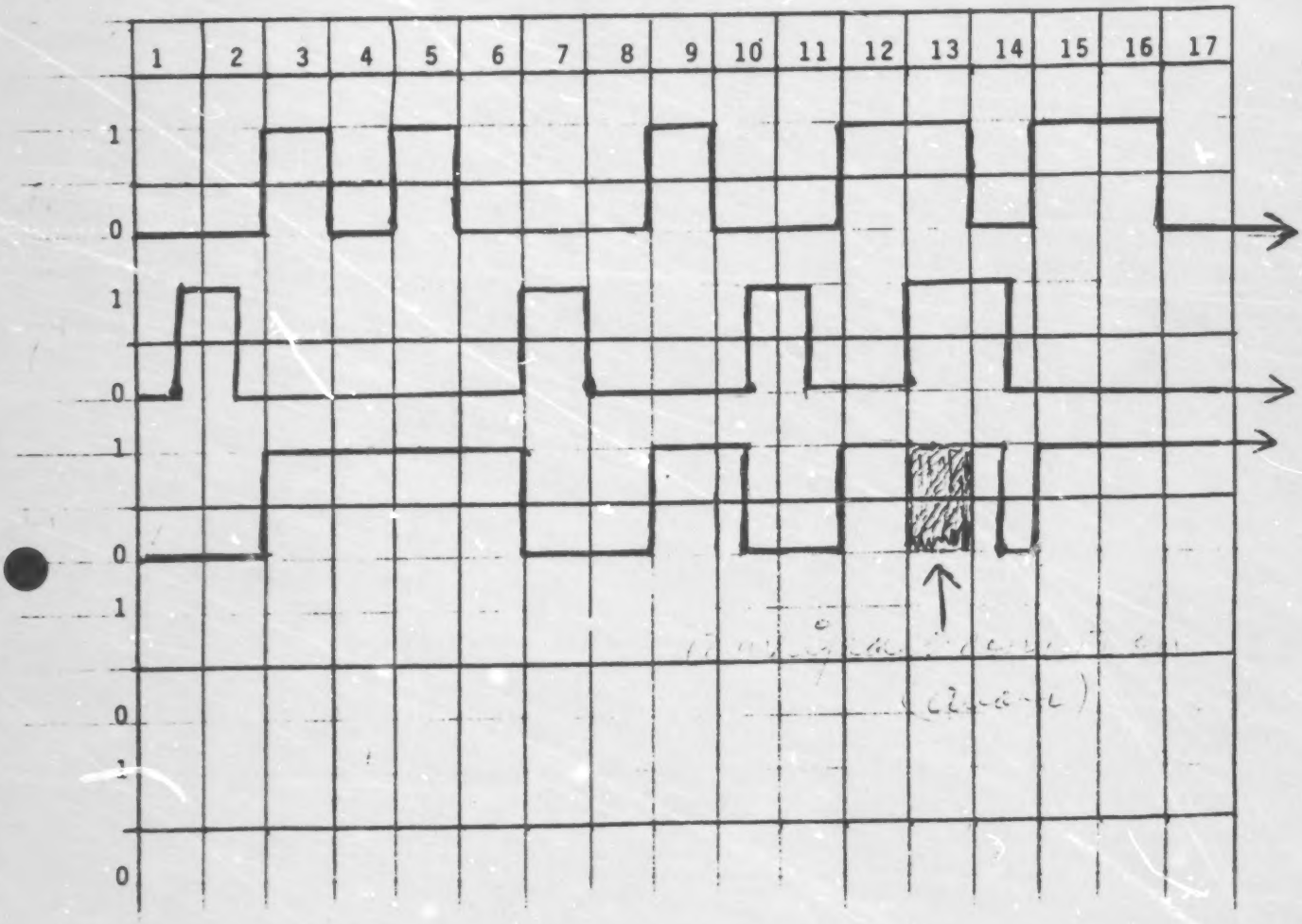
f) When both inputs are high the output is not predictable - of no value.

III-16 i.



Assume initial condition is clear and A and B are low.

NOTE: The changes from high to low on signals A and B do not change the output Q.


1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
1
0
1
0
1
0
1
0
1
0

III-17 1. a) most significant bit
b) least significant bit
c) latches connected in series so that the output of A becomes the input for B and so on

III-18 1. The LEDs will simply reflect the binary codes for the first 10 decimal numbers - beginning at zero.

0000 - 0
0001 - 1
0010 - 2
0011 - 3
0100 - 4
0101 - 5
0110 - 6
0111 - 7
1000 - 8
1001 - 9

LED stands for light emitting diode

OP.IV. INVESTMENTS (STOCKS, BONDS, REAL ESTATE)

In this section it is suggested that the teacher provide the students with a copy of an up-to-date report of a stock exchange. The teacher can then choose appropriate stocks to fill in the blanks in the questions that follow. Once the teacher does this, the student can then answer the questions.

A-1

1. What were the sales of _____ for the day?
2. What was the highest price that _____ sold for during the day?
3. What was the lowest price that _____ sold for during the day?
4. What was the closing price of _____ for the day?
5. _____ closed at _____ for the day. What was the closing price for the previous day?
6. _____ closed at _____ for the day. What was the closing price for the previous day?
7. What was the closing price of _____ for the day?
8. What was the lowest price that _____ sold for during the day?
9. What was the highest price that _____ sold for during the day?
10. What were the sales of _____ for the day?

A-2

Calculate the cost of purchasing the stocks listed below:

	Name of Stock	No. of Shares	Purchased At
1.		200	High
2.		1000	Low
3.		500	Close
4.		300	Low
5.		50	Close
6.		400	High
7.		300	Low
8.		150	Close
9.		1300	High
10.		1500	Close

11. R. Williams purchased 400 shares of _____ stock at the high of the day and 500 shares of _____ at the close of the day. What was the total cost of Williams' purchase?

12. J. Zeghers purchases 1000 shares of _____ at the high price of the day. How much would have been saved if the purchase had been made at the low price of the day instead?

A-3 Below is a schedule of commission rates that has been used by stockbrokers. Use this schedule to answer the questions that follow.

BASE RATES OF COMMISSION
FOR ORDERS UP TO AND INCLUDING $20,000

a). For SHARES selling under $14.00 the COMMISSION is 2.5% of the value of the ORDER.

b). For SHARES selling at $14.00 and up to, and including, $30.00 the COMMISSION is 22.575 CENTS PER SHARE PLUS 0.8875% of the value of the ORDER.

c). For SHARES selling over $30.00 the COMMISSION is 1.64% of the value of the ORDER.

MINIMUM COMMISSION

Orders under $10.00	Discretion of the broker.
Orders of $10-$50.00	$2.00
Orders over $50.00	$5.00

	Stock Name	No. of Shares	Selling Price	Commission
1.	Abitibi 10	200	$35	
2.	Argus	500	$8 1/4	
3.	Brascan A	400	$16 1/2	
4.	Brunswk	300	$7 7/8	
5.	Camel Og	1000	$310	
6.	Ceres Res	500	$160	
7.	C Rambler	700	$470	
8.	C Falcon C	3000	$19 1/2	
9.	Echo 1987 W	400	$135	
10.	Gaz Metr	4000	$35 3/8	

11. John Jakes purchased 2000 shares of Finning BF Stock at a price of $400. Find the commission that was paid on the transaction.

A-4 Using the Schedule of Commission Rates calculate the total cost of purchasing the following stocks.

	Stock	No. of Shares	Selling Price	Commission	Total Cost
1.	GMC	900	$57 1/2		
2.	Genstar L	2000	$12 3/4		
3.	F Cty Tn C	400	$15 3/8		
4.	Corby VT	300	$22 3/4		
5.	Cominco	4000	$35 1/4		
6.	Con Distrb	200	$ 6 3/4		
7.	Coho AF	1600	360		

8. F. Simmons purchased 250 shares of CIL at a price of $20 3/4. What was the total cost of the purchase?

9. J. Jones purchased 1400 shares of Cableshar at $5 7/8 and 1200 shares of CAD FW at $ 6 7/8. What was the total cost of the purchase?

10. F. Fitzimmons purchased 1000 shares of BCP at $30 5/8. What was the total cost of the purchase?

A-5 Calculate the net proceeds from selling the stocks listed below.

	Stock	No. of Shares Sold	Selling Price	Gross Proceeds	Commission	Net Proceeds
1.	Algoma St.	400	$26 1/2			
2.	Anthes	1000	92			
3.	BCPH	500	$31 1/2			
4.	Fed Ind A	1400	$11 1/2			
5.	Ford CNDA	1300	$32			
6.	Con Glass	150	$14 1/8			

7. D. Edwards decided to sell 3000 shares of F Calgary stock that had been bought two months previously. The selling price was $120. What were the net proceeds of the transaction?

8. E. Tolbert had purchased 200 shares of Chrysler stock at $8 1/2. Two months later Tolbert sold the stock when it was selling for $8 3/4. What were the net proceeds of the sale?

9. J. Walters decided to sell 300 GAZ 5.40 P stocks when the stock was selling at $66. What were the net proceeds of the sale?

10. The executor of the late J. Baines estate decides to sell shares purchased by the deceased. The executor sells 200 shares of FCA Intl stock at $8 and 250 shares of Extndcar stock at $8 1/2. What would be the net proceeds of the sale of stock to the estate?

A-6

1. M. Draper purchased 100 shares of Ang CT 4 1/2 stock at a price of $15. Two months later the stock was sold at $13 1/2. Calculate the capital gain or loss from the sale of stock.

2. W. Watt purchased 200 shares of Agra Ind A stock at a price of $8 3/8. Watt decided to purchase real estate with her investment money and sold the stock for $12 1/4. Calculate the capital gain or loss incurred while the stock was held by Watt.

3. A. Kaminski decided to sell 1000 shares of CDC 1980 stock that had been left to her by her grandfather's estate. She sold the stock at a price of $13 1/4. Her grandfather had originally purchased the stock at a price of $11 1/8. Calculate the capital gain or loss between the time her grandfather bought the stock and the time Kaminski sold it.

4. R. Monnin sold 400 shares of CDC PR B stock at a price of $84. The stock had been purchased earlier in the day at a price of $81. Calculate the capital gain or loss made by Monnin's transactions during that day.

5. Is it possible to incur a capital loss on the sale of stock even if the selling price is higher than the purchase price? Why?

6. H. Hold purchased 100 shares of Alt Energy at a price of $12 1/8. Later the shares were sold at a price of $12 1/4. Calculate the capital gain or loss on the transaction.

A-7

Find the total divident that the owners of the following common stock would receive:

	No. of Shares	Dividend and Period	Total Dividends for the year
1.	100	30¢ quarterly	
2.	250	50¢ semi-annually	
3.	1000	$1.20 annually	
4.	300	22¢ quarterly	
5.	400	38¢ quarterly	
6.	500	$1.10 semi-annually	
7.	200	$3.05 annually	
8.	100	$1.50 annually	
9.	150	40¢ quarterly	
10.	250	25¢ quarterly	

If you own preferred shares the dividend may be expressed as a percent rate of the par value.

Example: a $100 par value preferred stock paying a 5% dividend would pay: $100 x 5% = $5.00

Note that preferred dividends are always based on the par value and never the market value.

Find the total dividend that the owners of the following preferred stock would receive:

	No. of Shares	Par Value	Percent Rate of Dividend	Total Dividend
11.	75	$100	8%	
12.	150	$ 50	5 1/2 %	
13.	1000	$ 25	11%	
14.	800	$ 30	9%	
15.	2000	$ 20	6 3/4 %	
16.	300	$100	5 3/4 %	

A-8 Find the rate of return of the following stocks:

	Total Cost Per Share	Yearly Dividend	Percent Rate of Return
1.	$36 1/8	$2.75	
2.	$14 1/4	$1.60	
3.	$8 1/2	$0.65	
4.	360	$0.18	

5. Dwayne Ennis purchased 100 shares of GMC at $57 1/2 plus commission. The stock pays semi-annual dividends of $2.25 per share. What is the percent rate of return on his investment.

6. Donna Vennard purchased 200 shares of CAMEL OG stock at 310 plus commission. The stock pays quarterly dividends of 12 cents per share. What is the percent rate of return on her investment?

7. C. Bowman purchased 300 shares of BCP preferred stock at a price of $2[illegible] 5/8 plus commission. The stock pays a yearly dividend of 11% based on a par value of $25.00. What is the percent rate of return on this investment?

8. D. Metcalf purchased 300 shares of CON DISTRB common stock at a price of $6 3/4 plus commission. The stock pays a quarterly dividend of 18 cents per share. What is the percent rate of return on this investment?

9. C. Cuddy purchased 200 shares of EXTENDCAR stock at $8 1/2 plus commission and 150 shares of Elctham stock at $5 5/8 plus commission. The EXTENDCAR stock pays a quarterly dividend of 25 cents a share and the Elctham stock pays a semi-annual dividend of 30 cents a share. What is the percent rate of return on the total investment?

B Bonds

B-1 1. Compute the market price in dollars and cents of a $1000 bond if the bond was selling at the following quoted prices:

a) 98
b) 96 3/4
c) 102 1/8
d) 103 1/8
e) 86 3/8
f) 99 3/4
g) 108
h) 97 1/4

2. What would you pay for twenty bonds if you purchased the bonds at 93 1/4? The bonds have a par value of $1000.

Find the amount invested in the following bond acquisitions:

	No. of Bonds	Par Value	Quoted Price	Total Invested
3.	4	$1000	96 1/8	
4.	10	$ 500	94 1/4	
5.	8	$1000	102 1/8	
6.	5	$1000	107 3/8	
7.	6	$ 500	97 1/2	

B-2 Questions in this section will assume a commission rate of 1/2% on the value of the purchase.

Example: What would you pay for a $1000 bond selling at 98 1/2 plus commission.

Solution: Cost of 1 bond $1000 x 98 1/2 = $985
Commission $985 x .5% = $4.93
Total Cost $985 + $4.93 = $989.93

Calculate the total amount invested if you purchased the following bonds:

	No. of Bonds	Par Value	Quoted Price	Cost	Commission	Total Investment
1.	10	$1000	94 1/2			
2.	6	$1000	86 1/8			
3.	3	$ 500	101 3/4			
4.	7	$1000	92 1/4			
5.	12	$1000	104 1/2			

6. Tom Landry purchased five $1000 bonds at 97 1/2 plus commission. What would be the amount of his investment?

7. Phyllis George purchased six $1000 bonds at 101 1/8 plus commission and five $500 bonds at 93 3/4 plus commission. Find the total amount of her investment.

B-3 Calculate the total bond income for each of the following:

	No. of Bonds	Par Value	Interest Rate	Total Bond Income
1.	6	$1000	8 1/2 %	
2.	7	$1000	10 1/4 %	
3.	10	$1000	12 1/2 %	
4.	3	$ 500	13 1/8 %	

5. What will be the annual income from twelve bonds having a par value of $1000 and paying 9 1/2 % interest?

6. Joe Adams purchased five $1000 bonds at 96 1/2 which pay interest at the rate of 8 1/2 %. What will be th, annual income?

7. How much would each interest payment be on a $1000 bond paying 9% interest if the interest is paid semi-annually on August 1 and February 1?

For each of the following find the total investment and the total annual income.

NOTE: Don't forget to include commission of 1/2 % in your calculations.

	No. of Bonds	Par Value	Quoted Price	Total Invesement	Interest Rate	Annual Income
8.	4	$1000	88 1/2		9%	
9.	6	$1000	94 1/8		10%	
10.	10	$ 500	104 1/2		14%	
11.	12	$1000	84 1/4		7 1/2 %	
12.	8	$1000	93 1/8		9 1/2 %	

B-4 For each bond listed below, calculate the percent rate of return on investment.

	Par Value	Interest Rate	Quoted Price	Commission
1.	$1000	9%	96 1/8	.5%
2.	$1000	8 1/2 %	92 1/4	.5%
3.	$ 500	11%	99 1/2	.5%
4.	$ 500	12 1/4%	102 1/4	.5%
5.	$1000	7 1/8 %	88 1/2	.5%
6.	$1000	10 1/4 %	98 1/4	.5%

7. A $1000 bond, bearing interest at 13% was purchased at 104 1/2, plus commission. What is the percent rate of return on this investment?

8. K. Smith was trying to decide between two types of bonds. One bond was a $1000, 8% bond selling at 92 1/2 plus commission. The other bond was a $1000, 10% bond selling at 98 1/4 plus commission. Which bond has the higher percent rate of return on investment?

9. What is the percent rate of return on a $1000 bond purchased at 67 1/4 plus commission if the bond paid interest of 4 1/2 %?

10. What is the percent rate of return on a $1000 bond purchased at 105 1/2 plus commission if the bond paid interest of 13 1/4%?

C Real Estate

C-1 1. Wilson bought a house for $46 000 paying $14 000 down and assumed an existing 10% mortgage for the remainder. During the first year $425 a month was received for rent. Expenses for the year included:
. Interest at 10% on the amount of the mortgage
. 2% depreciation on the house
. Taxes, repairs, insurance and other expenses totaled $925
What was the net income from owning the house?

2. Smith decided to invest in real estate. She purchased both halves of a duplex for $84 000. During the year each side was rented out for $440 a month. Smith had a down payment of $31 000 and took out a mortgage at 15% for the balance. During the year the following expenses were incurred:
. Interest at 15% on the mortgage
. 1.5% depreciation on the house
. Taxes, insurance and other items $2100
Calculate Smith's net income from owning this duplex.

3. J. Snead bought a house for $52 000 paying $16 000 down and taking out a 14% morrgage on the remaining balance. Snead was able to rent the house for $475 a month. Expenses for the year included:
   . Interest at 14% on the mortgage
   . 3% depreciation on the house
   . Taxes of $760.00
   . Insurance of $151.00
   What was the net income from owning the house.

C-2

1. Calculate the rate of return on investment in problems 1, 2 and 3 of the previous section.

2. Given the above rates of return why would a person invest money in real estate?

Review Part A

Give the student a copy of a Stock Exchange. The teacher can then choose appropriate stocks to fill in the blanks in the questions that follow:

1. What were the sales of _____ during the day?

2. Shares of _____ closed at _____ today. This was an increase of _____ from yesterday's closing. What was yesterday's closing?

3. What was the lowest price that _____ sold for during the day?

4. What was the highest price that _____ sold for during the day?

5. What was the closing price of _____ for the day?

6. Calculate the total cost of purchasing 1000 shares of Corn Lake stock at a price of 15 plus commission.

7. Calculate the total cost of purchasing 500 shares Genstar 2.35 stock at a price of $17 5/8 plus commission.

8. Calculate the total cost of purchasing 400 shares of CDC pr B stock at a price of $85 1/2 plus commission.

9. Find the net proceeds from the sale of 100 shares of Alum 3rd p stock at a price of $20 less commission.

10. A person bought 1000 shares of Budd Can stock at a price of 430. Two months later the stock sold for $5.60. Calculate the capital gain or loss from the sale of stock.

11. What yearly income will a person receive from common stock that pays a dividend of 60 cents per share quarterly if they own 200 shares.

12. John Smalley purchased 100 preferred shares. The shares had a par value of $50.00 and had a dividend of 10%. Calculate the total dividend received by Smalley.

13. Butch Norman purchased 200 shares of CON Gas B stock at a price of $38.00 plus commission. The stock pays a semi-annual dividend of $1.60. What is the percent rate of return on this investment?

Review Part B

1. What would you pay for a $1000 bond selling at 86 1/2 plus commission of 1/2 % on the value of the purchase.

2. Roger Staubach purchased six $1000 bonds at 97 1/4 plus commission. What would be the amount of his investment?

3. Norm Turner purchased seven $1000 bonds at 93 1/2 plus commission and five $1000 bonds at 105 1/4 plus commission. What would be the amount of his investment?

4. John Sims purchased five $1000 bonds at 97 1/4 which pay interest at the rate of 9 1/2 %. What will be the annual income?

5. A $1000 bond, bearing interest at 14% was purchased at 103 1/2 plus commission. What is the percent rate of return on this investment?

6. A $1000 bond, bearing interest at 8 1/4 % was purchased at 94 1/8 plus commission. What is the percent rate of return on this investment?

Review Part C

1. Nettles decided to purchase a boarding-house. The boarding house cost $80 000. Nettles made a down payment of $20 000 and took out a $60 000 mortgage at 12% on the balance. The boarding-house had ten little suites. Each of these suites was rented out for $130 a month. Expenses for the year were:
   . Interest at 12% on the mortgage
   . Depreciation at 1% of the cost of the property
   . Taxes, insurance, and other expenses $2560
   Find the percent rate of return on this investment.

Overall Review

1. M. Barber purchased 1000 shares of BCRIC stock at a price of 265. Calculate the commission that was paid on the transaction.

2. D. Casper purchased 500 shares of BCP 10.16 preferred stock at a price of $20 5/8 plus commission. What was the total cost of the purchase?

3. K. Stabler decided to sell 1000 shares of CON Gas B stock at a price of $38. What were the net proceeds of the sale?

4. G. Neilsen purchased 300 shares of Chieftain D stock at a price of $18 5/8. Three months later he decided to sell the stock when its price had fallen to $13 5/8. Calculate the capital gain or loss from owning this stock.

5. What would be the annual dividend from owning 200 shares of common stock that pay a 30 cent quarterly dividend per share?

6. What would be your total dividend if you owned 100 preferred shares with a par value of $15 if the shares paid a dividend of 8%?

7. L. Crocker purchased 200 shares of CCL Ind stock at $5 1/4 plus commission. The stock pays semi-annual dividends of 30 cents per share. What is the percent rate of return on investment?

8. J. Thompson purchased six $1000 bonds at 98 1/2 plus commission. What would be the amount of the investment?

9. D. Parker purchased twelve $1000 bonds at 89 1/2 which pay interest at the rate of 8 1/2 %. What will be the annual income?

10. A $1000 bond, bearing interest at 12% was purchased at 95 1/4 plus commission. What is the percent rate of return on this investment?

11. For $33 000 Easler can buy a house which rents for $405 a month. Taxes, insurance, and miscellaneous expenses average $1050 annually. Depreciation and repairs are estimated at $1510 a year. If Easler pays $33 000 cash for the property calculate the percent rate of net income earned from this investment.

Answer Key

A-1 Answers will vary depending upon the type of stock that is chosen by the
A-2 teacher.

A-3

1. Cost of the stock: 200 x $35 = $7000.00, because the share is selling for over $30 the commission is 1.64% of the value of the order.
   Therefore, Commission $7000 x 1.64% = $114.80

2. Cost of the stock: 500 x $8.25 = $4125.00, because the share is selling for under $14 the commission is 2.5% of the value of the order.
   Therefore, Commission $4125 x 2.5% = $103.13

3. Cost of the stock: 400 x $16.50 = $6600.00, because the share is selling over $14 and under $30 the commission is 22.575 cents per share plus 0.8875% of the value of the order.
   Therefore, Commission is made up of 2 parts: commission on shares: 400 x .22575 = $90.30; commission on value of order: $6600 x 0.8875% = $58.58 - Total Commission: $90.30 + 58.58 = $148.88.

4. $59.06
5. $77.50
6. $20.00
7. $82.25
8. $1196.44
9. $13.50
10. $2320.60
11. $200

A-4

1. Cost of the stock: $57.50 x 900 = $51 750, because the share is selling for more than $30 the commission is 1.64% of the value of the order.
   Therefore, Commission: $51 750 x 1.64% = $848.70
   Total Cost of Purchase: $51 750 + $848.70 = $52 598.70

2. Cost of the stock: $12.75 x 2000 = $25 500.00, because the share is selling for under $14 the commission is 2.5% of the value of the order.
   Therefore, Commission: $25 500 x 2.5% = $637.50
   Total Cost of Purchase: $25 500 + 637.50 = $26 137.50

3. Cost of the stock: $15.375 x 400 = $6150, because the share is selling over $14 and under $30, the commission is 22.575 cents per share plus 0.8875% of the value of the order.
   Commission: 400 x .22575 = $90.30 + $6150 x 0.8875% = $54.58
   Total Commission: $90.30 + 54.58 = $144.88
   Total Cost: $6150 + 144.88 = $6294.88

4. $6953.30
5. $143 312.40
6. $1383.75
7. $5904.00
8. $5287.98
9. $16 886.88
10. $31 127.25

A-5

1. Gross Proceeds: $26.50 x 400 = $10 600, because the shares are selling over $14 and under $30 the commission is 22.575 cents per share plus 0.8875% of the value of the order.
   Commission: 400 x $0.22575 = $90.30 + $10 600 x 0.8875% = $94.08
   Total Commission: $94.08 + 90.30 = $184.38
   Net Proceeds: $10 600 - 184.38 = $10 415.62

2. Gross Proceeds: $0.92 x 1000 = $920.00, because the shares are selling for under $14 the commission rate is 2.5% of the value of the order.
   Commission: 920.00 x 2.5% = $23.00
   Net Proceeds: $920.00 = $23.00 = $897.00

3. Gross Proceeds: $31.50 x 500 = $15 750, because the shares are selling for over $30 the commission rate is 1.64% of the value of the order.
   Commission: $15.750 x 1.64% = $258.30
   Net Proceeds: $15 750 - $258.30 = $15 491.70

4. $15 697.50
5. $40 917.76
6. $2066.09
7. $3510
8. $1706.25
9. $19 475.28
10. $3631.88

A-6

1. Purchase Price:
   Cost: 100 x $15 - $1500
   Commission: 100 x $0.22575 = $22.58 + $1500 x 0.8875% = $13.31
   Total Commission: $22.58 + 13.31 = $35.89
   Total Cost: 1500 + 35.89 = $1535.89

   Selling Price:
   Gross Proceeds: 100 x $13.50 = $1350
   Commission: $1350 x 2.5% = $33.75
   Net Proceeds: $1350 - $33.75 = $1316.25

   Capital Loss: $1535.89 - 1316.25 = $219.64

2. Purchase Price:
   Cost: $200 x $8.375 = $1675.00
   Commission: $1675 x 2.5% = $41.88
   Total Cost: $1675 + 41.88 = $1716.88

   Selling Price:
   Gross Proceeds: 200 x $12.25 = $2450
   Commission: $2450 x 2.5% = $61.25
   Net Proceeds: $2450 - $61.25 = $2388.75

   Capital Gain: $2388.75 - 1716.88 = $671.87

3. Capital Gain: $1515.62
4. Capital Gain: $117.60
5. Yes, because of the commission involved
6. Capital Loss: $48.43

A-7

1. Annual dividend: .30 x 4 = $1.20
   Total dividend: $1.20 x 100 = $120

2. Annual dividend: .50 x 2 = $1.00
   Total dividend: $1.00 x 250 = $250

3. $1200
4. $264
5. $608
6. $1100
7. $610
8. $150
9. $240
10. $250

11. Dividend on 1 share: $100 x 8% = $8.00
    Total dividend: 75 x $8.00 = $600.00

12. Dividend on 1 share: $50 x 5.5% = $2.75
    Total dividend: 150 x $2.75 = $412.50

13. $2750
14. $2160
15. $2700
16. $1725

A-8

1. 7.6%
2. 11.2%
3. 7.6%
4. 5%

5. Cost of stock: $57.50 x 100 = $5750
   Commission: $5750 x 1.64% = $94.30
   Total Investment: $5750 + $94.30 = $5844.30

   Annual dividend per share: $2.25 x 2 = $4.50
   Total dividends: $4.50 x 100 = $450

   Using the percent formula A = R x B, we get $450 = R x $5844.30
   Solving for R we get a rate of return of 7.7%

6. 15.1%
7. 13.1%
8. 10.4%
9. 11.1%

B-1

1. a) $980
   b) $967.50
   c) $1021.25
   d) $1031.25
   e) $963.75
   f) $997.50
   g) $1080
   h) $972.50

2. $18 650
3. $3845
4. $4712.50
5. $8170
6. $5368.75
7. $2925

B-2

1. $9497.25
2. $5193.34
3. $1533.88
4. $6489.79
5. $12 602.70
6. $4899.38
7. $8453.31

B-3

1. $510
2. $717.50
3. $1250
4. $196.88
5. $1140
6. $412.50
7. $45

8. Price of 1 Bond: $1000 x 88 1/2 = $885
   Price of 4 Bonds: $885 x 4 = $3540
   Commission: $3540 x .5% = $17.70
   Total Investment: $3540 + $17.70 = $3557.70

   Income from 1 Bond: $1000 x 9% = $90
   Interest from 4 Bonds: $90 x 4 = $360.00

9. $5675.74, $600
10. $5251.13, $700
11. $10 160.55, $900
12. $7487.25, $760

B-4

1. Price of Bond: $1000 x 96.125% = $961.25
   Commission: $961.25 x .5% = $4.81
   Total Investment: $961.25 + $4.81 = $966.06

   Interest on Bond: $1000 x 9% = $90

   Using A = R x B
   90 = R x 966.06
   90/966.06 = R
   Therefore, the rate of return is equal to 9.3%.

2. 9.2%
3. 11.1%
4. 11.9%
5. 8.0%
6. 10.4%
7. 12.4%
8. the second bond pays 10.1% while the first pays only 8.6%
9. 8.1%
10. 12.5%

C-1

1. Annual Gross Income: $425 x 12 = $5100
   Expenses:
   a) Interest: $32 000 x 10% = $3200
   b) Depreciation: $46 000 x 2% = $920
   c) All other: $925
      Total Expenses: $5045
   Net Income: $5100 - $5045 = $55.00

2. Annual Gross Income: $440 x 12 x 2 = $10 560
   Expenses:
   a) Interest: $53 000 x 15% = $7950
   b) Depreciation: $84 000 x 1.5% = $1260
   c) Other: $1200
      Total Expenses: $10 410
   Net Income: $10 560 - $10 410 = $150.00

3. Annual Gross Income: $475 x 12 = $5700
   Expenses:
   a) Interest: $36 000 x 14% = $5040
   b) Depreciation: $52 000 x 3% = $1560
   c) Taxes: $760
   d) Insurance: $151
   Total Expenses: $7511
   Net Loss: $7511 - 4700 = $1811

C-2

1. a) 0.4%
   b) 0.5%
   c) -11.3%
2. One reason might be an expected capital gain.

Review Part A

1 - 5. Answers will vary
6. $153.75
7. $9003.59
8. $34 760.88
9. $2040.33
10. $1052.50 Gain
11. $480
12. $500
13. 8.3%

Review Part B

1. $869.33
2. $5864.18
3. $11 866.54
4. $475.00
5. 13.5%
6. 8.7%

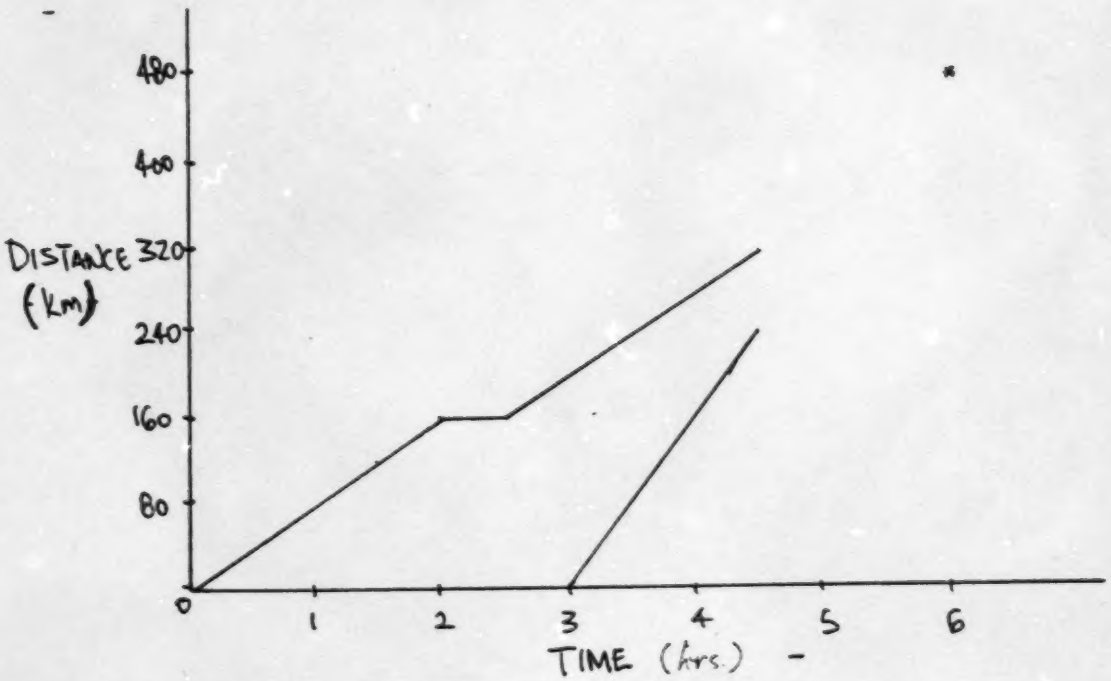
Review Part C

1. 25.2%

Overall Review

1. $66.25
2. $10 516.90
3. $37 376.80
4. $1819.51 Loss
5. $240
6. $120
7. 11.1%
8. $989.93
9. $1020
10. 12.5%
11. 7.0%

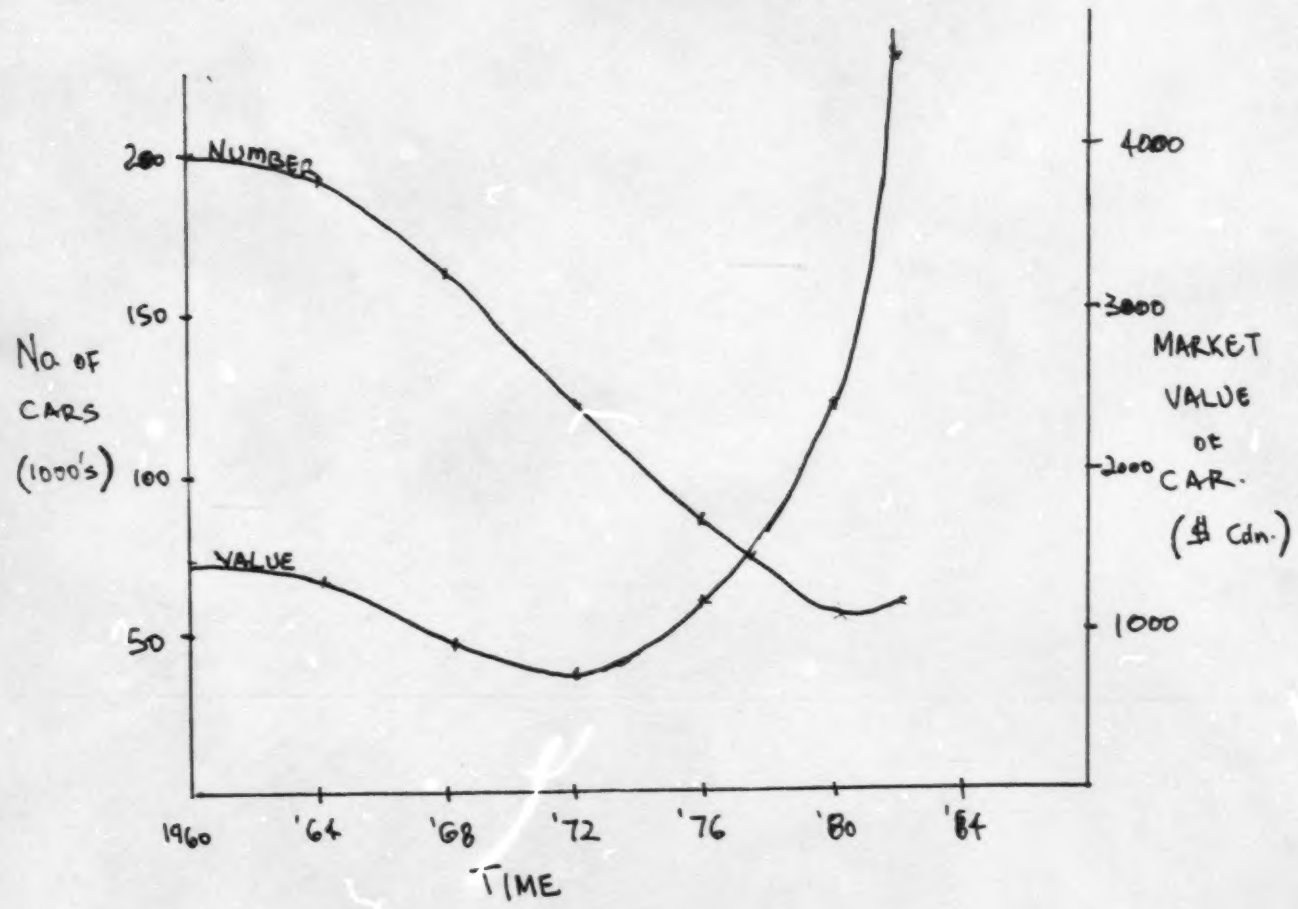
OP.V. INTERPRETATION OF GRAPHS (1/2 module)



A bus and an airplane have a journey of 480 km to be completed in 6 hours. The bus started at T_o, the airplane left 3 hours later. Answer the following questions.

a) How far had the bus gone after 1 1/2 hours? 4 hours?
b) After 4 1/2 hours of travel, which vehicle is travelling fastest? Prove your answer.
c) Assuming the speed of the bus and the plane remain constant, which will reach destination on time. Justify your answer graphically.
d) What is the speed of the airplane?
e) What is the speed of the bus from T_o to T_2? From $T_{2-1/2}$ to $T_{4-1/2}$?
f) How do we know the bus stopped?
g) What is the average speed of the bus for this travel shown?
h) What speed from $T_{4-1/2}$ to T_6 is necessary for the bus to reach its goal on time?
i) How is slope related to speed?
j) If the aircraft stopped for 1 hour, what speed would be necessary for it to reach its goal on time?

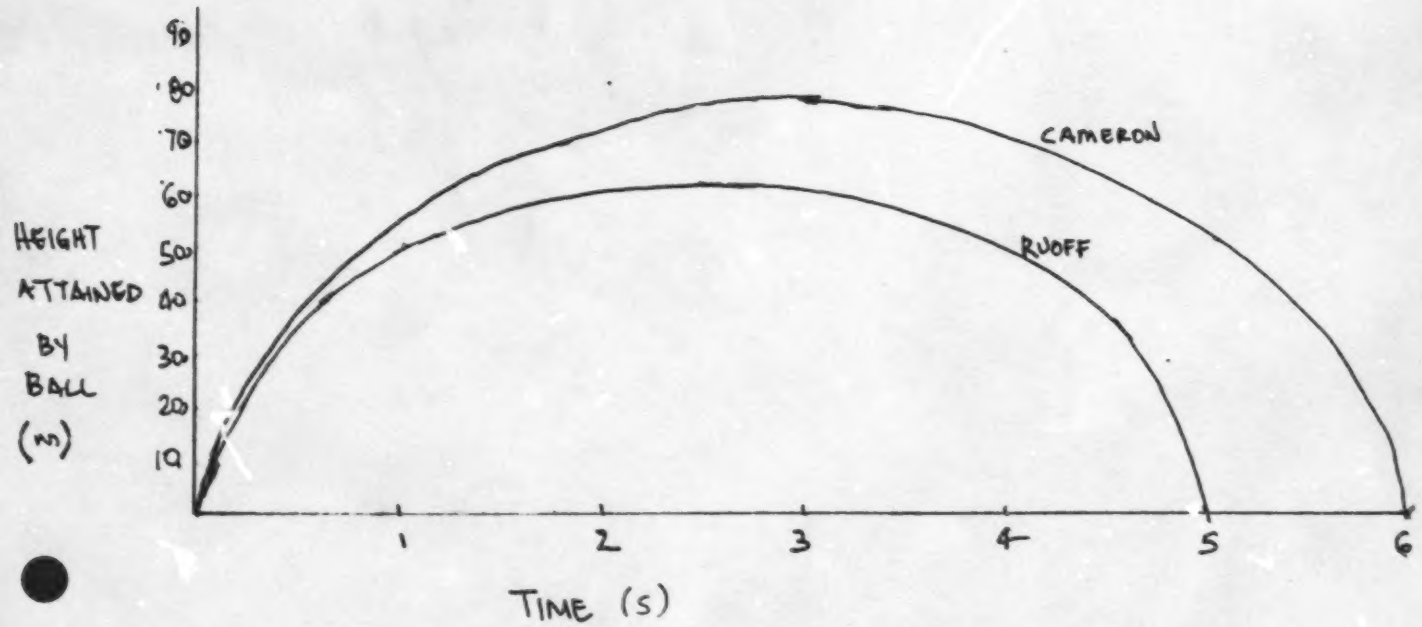
2.



The multiple line graph above indicates the number of existing 1960 M.G.'s and the market value of those automobiles over a 22 year period. Answer the following:

a) What was the lowest price for which an M.G. could be purchased?
b) What do you estimate is the price in 1982?
c) Why did the value begin to increase for an M.G. after 1972?
d) Can you explain why the number of cars increased after 1980 if the car was no longer produced?
e) Explain the phrase, "Hedge Against Inflation." Could a 1960 M.G. be a hedge against inflation?
f) Why do you think M.G.'s are no longer produced?
g) Can the 1980-82 price trend continue? Why?
h) Why was the car not considered to be more valuable until the mid-1970's?

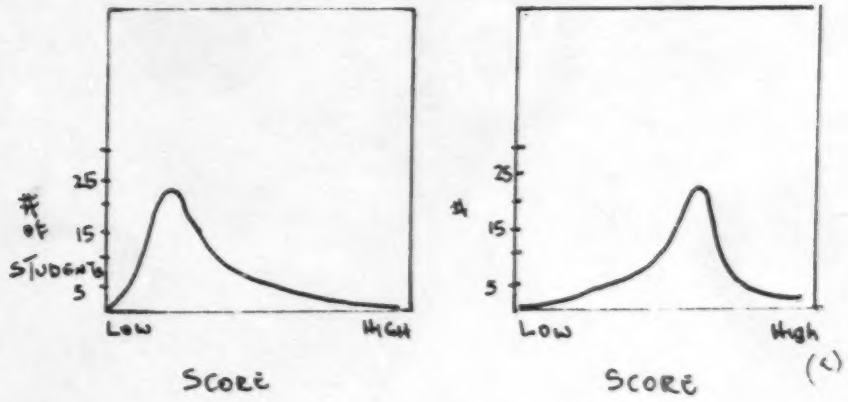
3.



The behaviour (vertical climb vs. time) of 2 kicked footballs is graphically shown above. The Cameron punt is in the air for 6 seconds, the Ruoff punt for only 5. Answer the following:

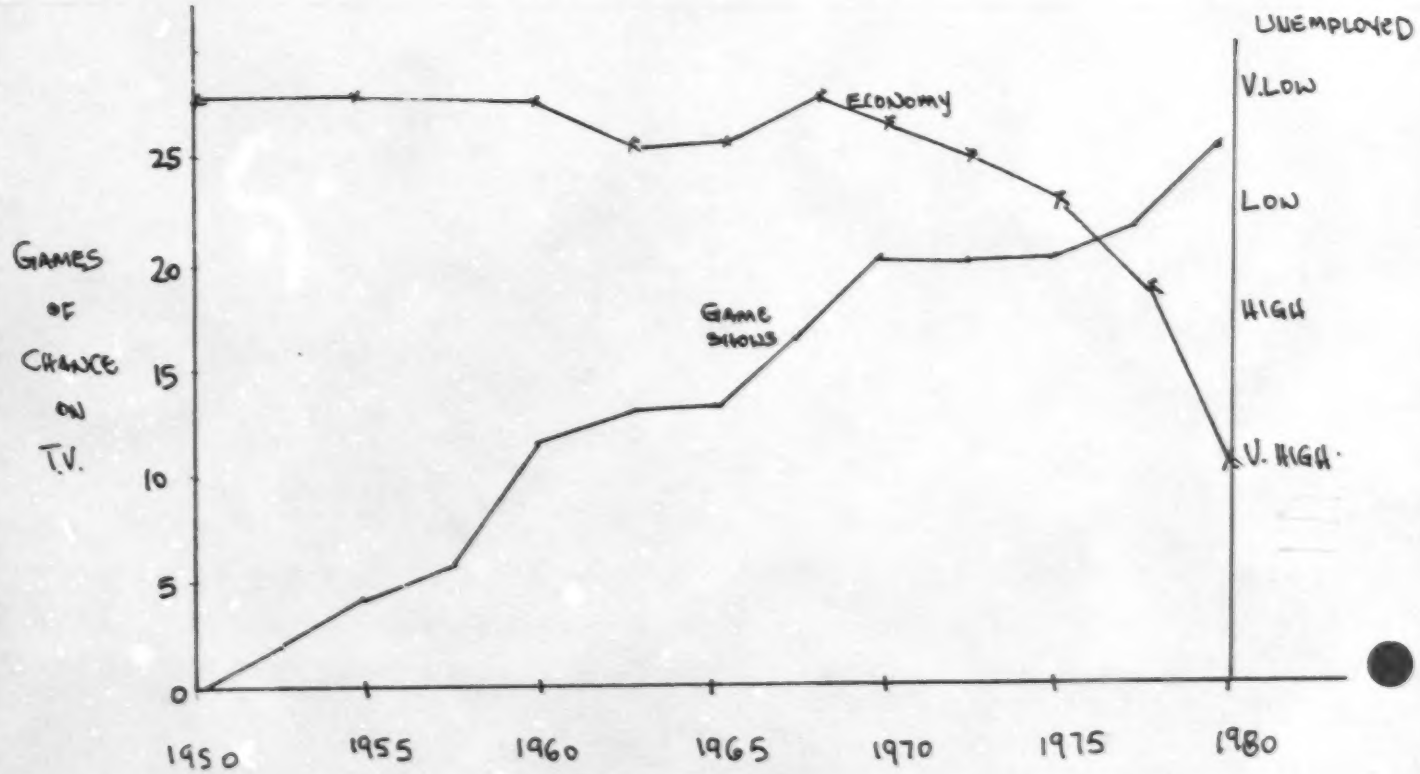
a) What is the difference in height attained by the 2 footballs?
b) Which football travelled faster when kicked? Justify your answer.
c) Show that each ball is slowing down in the first half of its journey, and accelerating in the last half.
d) What is the vertical velocity of the Cameron football at T_3? Of the Ruoff football at $T_{2.5}$?
e) Can the information above tell us the length of each kick?
f) After the half way point of the kick, each ball moves vertically according to free-fall physics, i.e., distance fallen = $1/2\ at^2$ where a = constant 9.8 and t = time in seconds.

4. Calculate the vertical drop per second of each ball. Can you calculate rates of acceleration?



These two distribution curves describe results of two spelling tests given the same class, one week apart. Interpret the graphs indicating what you believe accounts for the shape.

5.

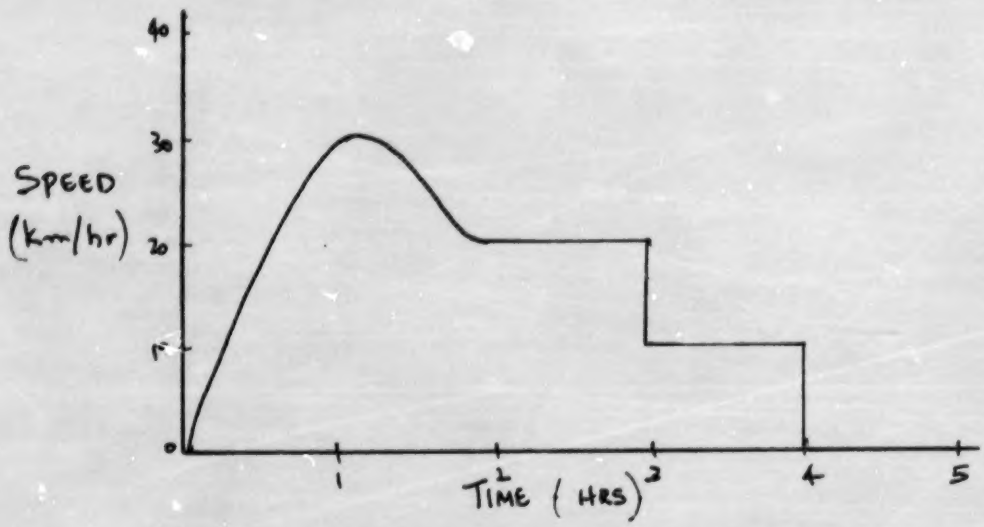


The preceeding line graph charts the number of game shows, games of chance, etc., available to viewers in a Canadian city, against the relative unemployment figures. From your analysis of the graph and knowledge of the period:

a) Why was the economy good in 1950?
b) Why were there few game shows on T.V. in 1950?
c) Suggest reasons why the unemployment increased in the early 1960's and game shows increased.
d) Suggest reasons for higher unemployment after 1970.
e) After 1975, what major, new trend occurred in games of chance?
f) State a possible generalized correlation between the number of unemployed and the number of games of chance.
g) Why was the economy shaken in the early 1960's?

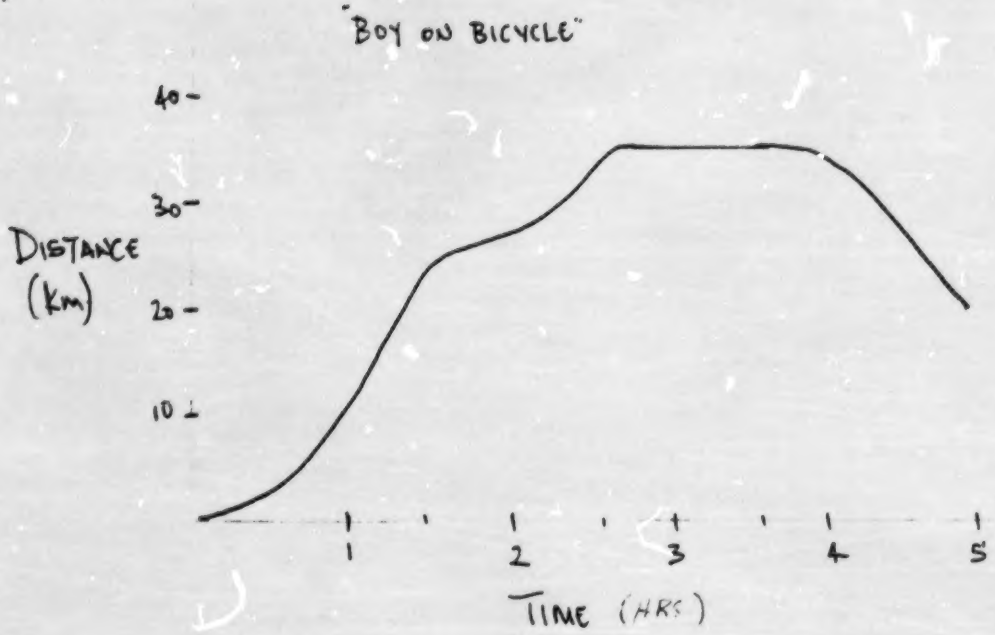
NOTE: The game of monopoly was created in the height of the depression.

6.



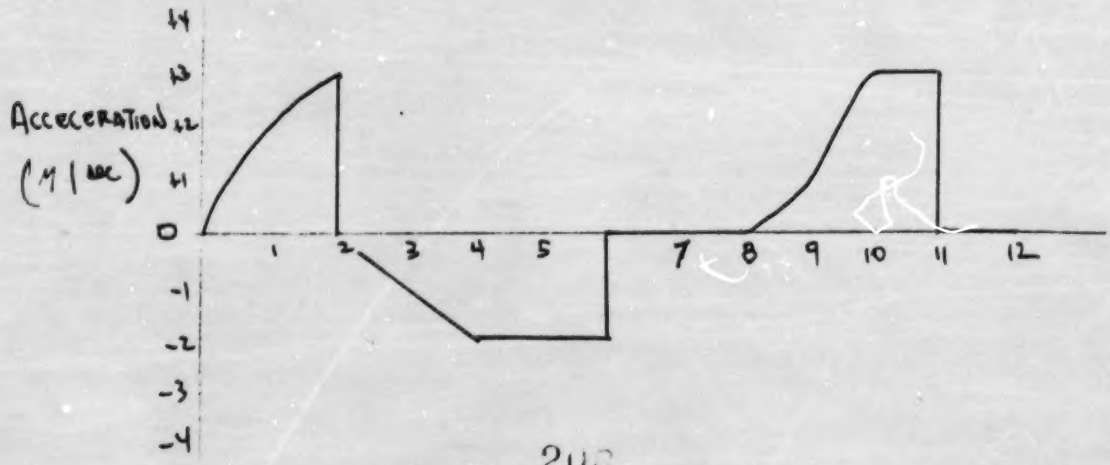
a) Estimate distance travelled.
b) Why did graph drop at T_4?
c) Where are the periods of braking?
d) Calculate rate of acceleration from T_1 to T_2.

7.



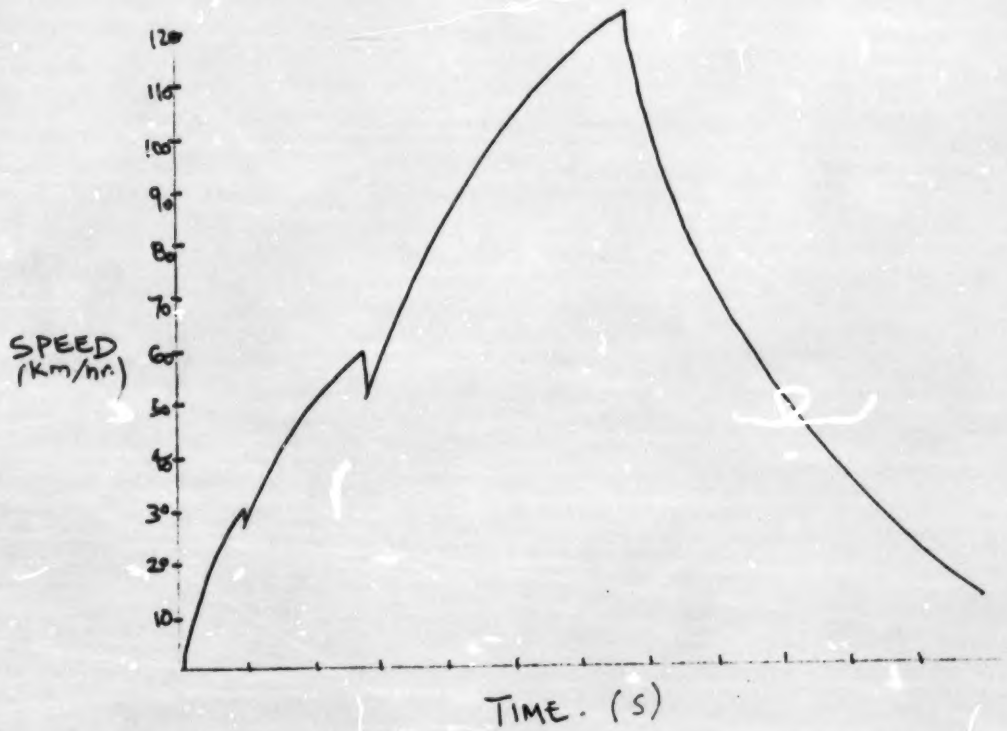
a) Describe what type of trip the boy is enjoying (consider road conditions, scenery, length of trip, etc.).

8.



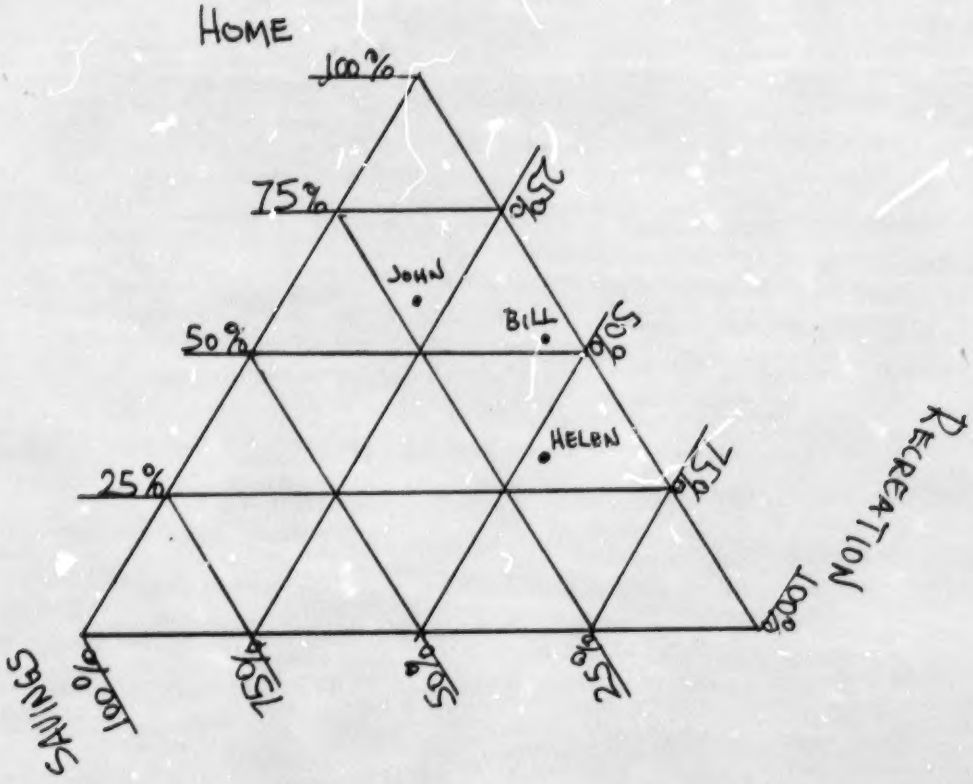
This graph represents acceleration, or the reverse, for a ball in a pinball game. Can you suggest what is happening to the ball?

9.



This graph represents a new, 4 speed automobile being tested. Can you suggest what happens?

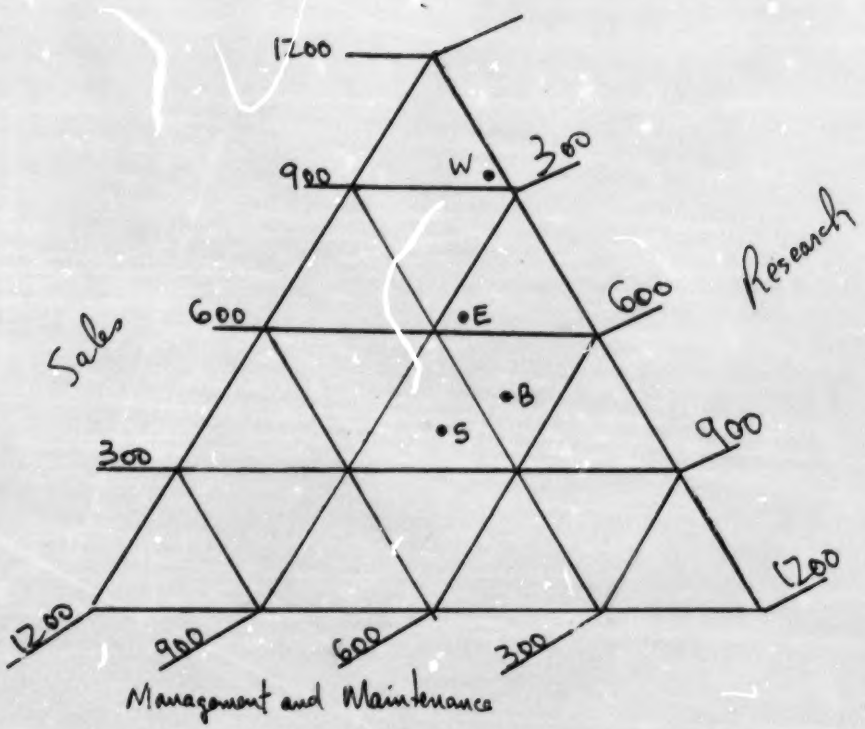
10.



The above graph indicates how 3 individuals spend net pay. Varying percent rates are spent on the home, recreation and savings.

a) What percent is spent by Helen on her home? savings?
b) What percent is spent by John on recreation? savings?
c) If Bill were to spend 25% on recreation, not touching home money, what is left for savings?
d) Without touching home money, if John spent notning on recreation, what percent would go to savings?
e) If Helen had to spend 60 percent of her pay on her home, keeping her savings the same, what could she spend on recreation? Would that be a greater or lesser percent than she has been paying?

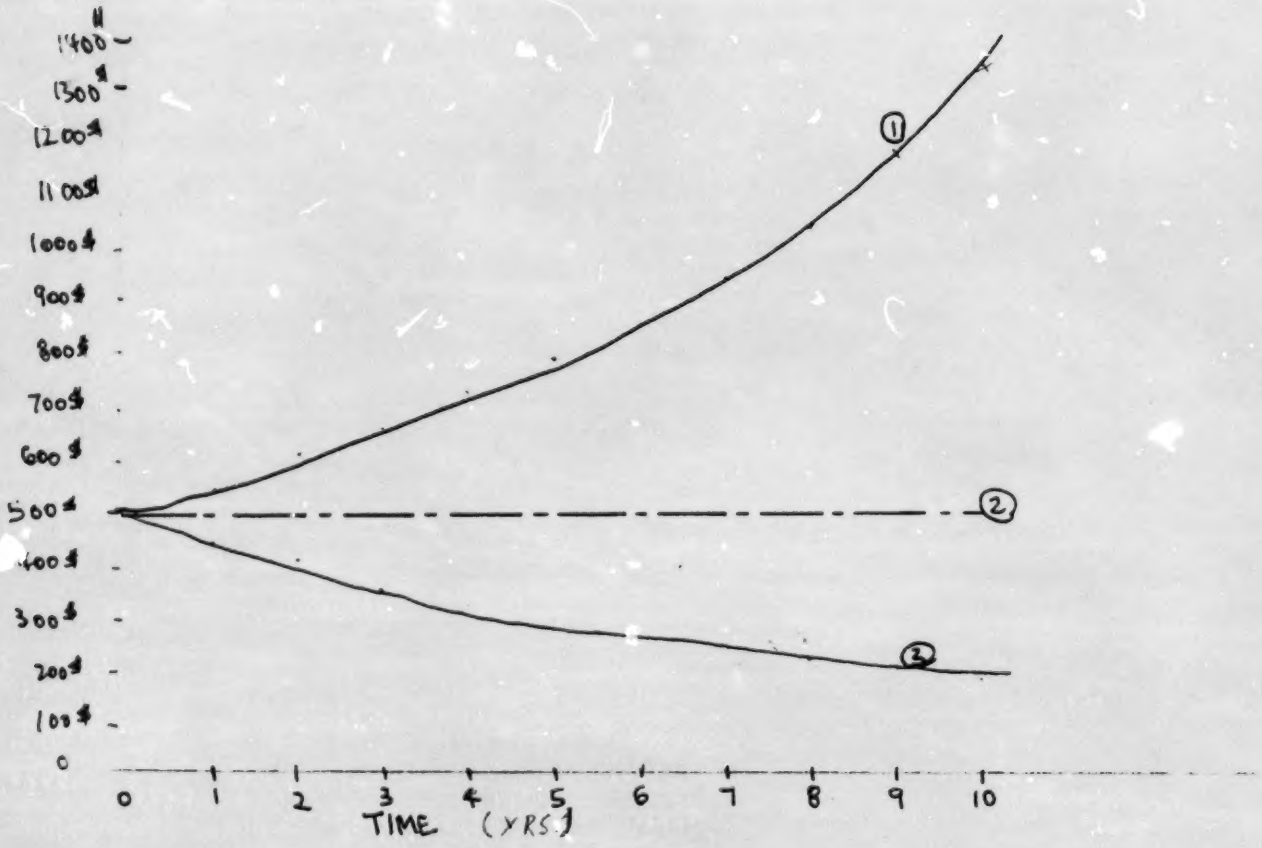
11.



The above graph represents how four major retail stores; Eaton's (E), The Bay (B), Sears (S), and Woolworths(W), employ in various departments (sales, research, management and maintenance).

a) How many people are employed in different departments by Sears? How many in each department?
b) Which store employs the greatest number of people?
c) How many more work in management and maintenance for Woolworths than for The Bay?
d) What two stores have equal numbers of people in management and maintenance?
e) How many are in sales at Eaton's?
f) How many are in research at Sears?

12.



The above graph indicates the results of investing $500 at 10 percent for 10 years (Line 1), and leaving $500 alone (Line 2) where no interest is earned; as well, an inflation rate of 10% has reduced buying power of the original $500 (Line 3).

a) What buying power has actually been lost over 10 years (between Lines 2 and 3)?
b) To what extent has the $500 "Earned" more money?
c) How much interest was earned after 3 years, 5 years?
d) What necessary information has not been included (i.e., can you detect any apparent illusion in this graph)?

OP.VI. QUADRATIC EQUATIONS (1/2 module)

1 A. Solve the following equations:

a) $5x^2 = 45$

b) $6x^2 = 96$

c) $7x^2 = 343$

d) $4x^2 - 36 = 0$

e) $16x^2 - 16 = 0$

f) $9x^2 - 4 = 0$

g) $25x^2 - 36 = 0$

h) $4x^2 - 20 = 0$

i) $4x^2 - 20 = 0$

j) $5x^2 - 75 = 0$

k) $8x^2 - 7 = 0$

l) $5m^2 - 6 = 0$

m) $-3x^2 + 23 = 0$

B. Solve the following quadratic equations, if possible. If there is no solution, indicate so.

a) $9x^2 - 1 = 0$

b) $\frac{1}{2}x^2 - 8 = 0$

c) $2x^2 - 8 = 0$

d) $5x^2 - 6 = 0$

e) $5x^2 + 6 = 0$

f) $4x^2 - 1 = 0$

g) $(2x + 1)(2x - 1) = 0$

h) $(x + 3)(2x - 6) = 0$

i) $x^2 + 4 = 0$

j) $5x^2 + 3 = 7$

k) $3x^2 - 4 = 1$

l) $2x^2 + 7 = 1$

m) $4x^2 + 9 = 9$

2 A. Arrange the following in the $ax^2 + bx + c = 0$ order and state the value of a, b, and c for each.

a) $2x^2 + 6x + 9 = 0$

b) $6x^2 - 7x + 16 = 0$

c) $x^2 + 8x + 32 = 0$

d) $x^2 + 5x + 1 = 0$

e) $x^2 + x - 1 = 0$

f) $-4x^2 - 8x + 9 = 0$

g) $3x^2 + 6x = 13$

h) $-4x^2 = 9x + 4$

i) $9 = 3x^2 - 7x$

B. Arrange the following in the order $ax^2 + bx + c = 0$ and state the value of a, b and c for each.

a) $5 - 8x^2 = 3x$
b) $3 + 7x - x^2 = 0$
c) $2 + 5x^2 = 8x$
d) $(3x + 1)(2x - 7) = 0$
e) $(8x + 3)(2x + 5) = 0$
f) $(x - 7)(2x - 1) = 0$
g) $3(2x^2 + 6x) = 9$
h) $(x + 2)(x - 3) = (2x + 4)(3x - 1)$
i) $(2 - 5)(2x + 1) = (x + 7)(x - 7)$
j) $3(x + 5) + 4(x^2 - 3x) = 4(3x + 6)$
k) $5x(x + 6) = 3x(2x - 8) + 8$

C. Arrange the following in the order $ax^2 + bx + c = 0$ and state the value of a, b and c for each.

a) $6x^2 + 3 = 0$
b) $5x^2 - 8x = 0$
c) $-3x^2 = 7$
d) $4x^2 = -9$
e) $(2x + 3)(2x - 3) = 0$
f) $3x(x + 2) = 2x(x - 5)$
g) $x(x + 9) = 4x(2x - 3)$
h) $2(x + 3) - 4x(2x - 1) = (x + 1)^2$
i) $(2x + 3)^2 = (2x - 7)^2$
j) $(x + 1)(x + 1) = x^2 - 7$
k) $3(x + 2)^2 = 3(x - 2)(x + 4)$

3 A. Use the quadratic formula to solve each of the following equations:

a) $6x^2 + 17x + 7 = 0$
b) $x^2 + 6x + 5 = 0$
c) $x^2 + 6x + 8 = 0$
d) $x^2 - 4x + 4 = 0$
e) $x^2 + 2x - 63 = 0$
f) $2x^2 - 3x - 14 = 0$
g) $6x^2 + x - 1 = 0$
h) $x^2 - 4x - 21 = 0$
i) $6x^2 + 19x - 7 = 0$
j) $10x^2 - 3x - 1 = 0$
k) $-6x^2 + 5x - 1 = 0$
l) $-5x^2 - 4x + 12 = 0$

B. Use the quadratic formula to solve each of the following equations:

a) $x^2 + 7x + 1 = 0$
b) $2x^2 - 6x - 1 = 0$
c) $3x^2 + 10x + 2 = 0$
d) $-4x^2 + 6x - 1 = 0$
e) $-7x^2 - 13x + 5 = 0$
f) $-3x^2 - 8x - 2 = 0$
g) $5x^2 + 2x - 3 = 0$
h) $-2x^2 + 1x + 4 = 0$
i) $7x^2 - 8x - 9 = 0$
j) $x^2 + x - 1/2 = 0$
k) $x^2 + 1/2x - 1/4 = 0$
l) $-1/2x^2 - 1/2x + 1/3 = 0$

C. Solve each of the following equations:

a) $x^2 - 8 = 0$

b) $3x^2 - 6 = 0$

c) $5x^2 - 50 = 0$

d) $-3x^2 + 25 = 0$

e) $4x^2 = 13$

f) $2x^2 - 3x = 0$

g) $x^2 + 4x = 0$

h) $x^2 - 6x = 0$

i) $1/2x^2 + 8x = 0$

j) $3x^2 = 7x$

k) $-2x^2 = 6x$

l) $1/4x^2 = 1/8x$

4 A. Write an equation for each of the following problems and then solve. Indicate what each variable represents.

a) A rectangle has a perimeter of 42 cm and an area of 104 cm^2. Find its dimensions.

b) Find two consecutive integers whose product is 182.

c) If a number is added to its square, the result is 462. Find the number.

d) The product of two consecutive even integers is 728. Find the integers.

e) What are the dimensions of a badminton court whose area is 98 m^2 and whose length is twice its width?

f) The width of a rectangle is 7 cm shorter than its length and 8 inches shorter than its diagonal. How long is the diagonal?

g) The height of an object, h, shot vertically upward with an initial velocity, v, is given by the formula $h = vt - 16t^2$, where t is the time in seconds. If an object is shot upward with an initial velocity of 96 m/sec, in how many seconds will it be (i) 144 m above the ground? (ii) 140 m above ground?

h) For a given right triangle, it is known that the hypoteneuse is 1 unit longer than one of the legs and 18 units longer than the other leg. How long are the sides of the triangle?

i) Three boys share the cutting of a lawn by beginning at the outer edge and travelling around the rectangular shaped lawn. (a) If the lawn measures 40 m by 90 m, how wide a strip must the first boy mow so that he does his 1/3 share? (b) How wide a strip must the second boy do to get his 1/3 share done?

j) A 5 cm square is cut from each corner of a square piece of tin, and then each side is folded up to form an open box which has a volume of 845 cm^3. Find the dimensions of the original piece of tin.

k) A rectangular piece of tin is 10 cm shorter than its length. A four inch square is cut from each corner, the sides are turned up to form an open box of volume 684 cm^3. Find the dimensions of the original piece of tin.

l) The sum of a number and its reciprocal is 4. Find the number.

m) Find the length of a side of a square if the number of square centimetres in its area is twice the number of centimetres of its perimeter.

n) Over the past 2 years the price of shirts has risen $6 each so that 4 less shirts can now be purchased for $144 than 2 years ago. What price do shirts sell for today?

Answer Key

1

A. a) $x = \pm 3$
b) $x = \pm 4$
c) $x = \pm 7$
d) $x = \pm 3$
e) $x = \pm 1$
f) $x = \pm 2/3 \doteq \pm 0.67$
g) $x = \pm 6/5 = \pm 1.2$
h) $x = \pm\sqrt{5} \doteq \pm 2.24$
i) $x = \pm 3/2 = \pm 1.5$
j) $x = \pm\sqrt{15} \doteq \pm 3.87$
k) $x = \pm\sqrt{7/8} \doteq \pm 0.94$
l) $m = \pm\sqrt{6/5} \doteq \pm 1.10$
m) $x = \pm\sqrt{23/3} \doteq \pm 2.77$

B. a) $x = +1/3 \doteq \pm 0.67$
b) $x = \pm 4$
c) $x = \pm 1/4 = \pm 0.25$
d) $x = \pm\sqrt{6/5} \doteq \pm 1.10$
e) no solution; imaginary
f) $x = \pm 1/2 = \pm 0.5$
g) $x = \pm 1/2 = \pm 0.5$
h) $x = +3$
i) no solution
j) $x = \pm\sqrt{4/5} \doteq \pm 0.89$
k) $x = \pm\sqrt{5/3} \doteq \pm 1.29$
l) no solution
m) $x = 0$

2

A. a) 2, 6, 9
b) 6, -7, 16
c) 1, 8, 32
d) 1, 5, 1
e) 1, 1, -1
f) -4, -8, 9
g) $3x^2 + 6x - 13 = 0$; 3, 6, -13
h) $4x^2 + 9x + 4 = 0$; 4, 9, 4
i) $3x^2 - 7x - 9 = 0$; 3, -7, -9

B. a) $8x^2 + 6x - 5 = 0$; 8, 6, -5
b) $x^2 - 7x - 3 = 0$; 1, -7, -3
c) $5x^2 - 8x + 2 = 0$; 5, -8, 2
d) $6x^2 - 19x - 7 = 0$; 6, -19, -7
e) $16x^2 + 46x + 15 = 0$; 16, 46, 7
f) $2x^2 - 15x + 7 = 0$; 2, -15, 7
g) $2x^2 + 6x - 3 = 0$; 2, 6, -3
h) $5x^2 + 11x + 2 = 0$; 5, 11, 2
i) $x^2 - 9x + 44 = 0$; 1, -9, 44
j) $4x^2 - 21x - 9 = 0$; 4, -21, -9
k) $x^2 - 54x + 8 = 0$; 1, -54, 8

C. a) $6x^2 + 0x + 3 = 0$; 6, 0, 3
b) $5x^2 + -8x + 0 = 0$; 5, -8, 0
c) $3x^2 + 0x + 7 = 0$; 3, 0, 7
d) $4x^2 + 0x + 9 = 0$; 4, 0, 9
e) $4x^2 + 0x - 9 = 0$; 4, 0, -9
f) $x^2 + 16x + 0 = 0$; 1, 16, 0
g) $7x^2 - 21x + 0 = 0$; 7, -21, 0
h) $9x^2 - 4x - 5 = 0$; 9, -4, -5
i) $0x^2 + 40x - 40 = 0$; 0, 40, -40
j) $0x^2 + 2x + 8 = 0$; 0, 2, 8
k) $0x^2 + 6x + 36 = 0$; 0, 6, 36

3 A. a) $\frac{1}{2}$
b) -1, -5
c) -2, -4
d) +2
e) 7, -9
f) 7/2, -2
g) 1/3, $\frac{1}{2}$
h) 7, -3
i) 1/3, -3 1/2
j) $\frac{1}{2}, -\frac{1}{5}$
k) 1/3, 1/2
l) -2, 6/5

B. a) -0.15, -6.85
b) $\frac{3}{2}, \frac{\pm\sqrt{11}}{2}$
c) -0.214, -3.12
d) $\frac{3}{4}, \frac{\pm\sqrt{5}}{4}$
e) -2.184, -0.327
f) $\frac{-4\pm\sqrt{10}}{3}$
g) 3/5, -1
h) -1.19, 1.69
i) $\frac{4}{7}, -\frac{36}{7}$
j) -1.366, 0.37
k) $-\frac{1}{4}, \frac{\pm\sqrt{5}}{4}$
l) -1.46, 0.46

C. a) $\pm 2\sqrt{2}$
b) ± 1.414
c) $\pm\sqrt{10}$
d) ± 2.885
e) $\frac{\pm\sqrt{13}}{2}$
f) 3/2, 0
g) -4, 0
h) 6, 0
i) -16, 0
j) 14/3, 0
k) -3, 0
l) 1/2, 0

4 a) $L^2 - 21L + 104 = 0$; L = 13, W = 8
b) $x^2 + x - 182 = 0$; 13, 14
c) $x^2 + x - 462 = 0$; 21, -22
d) $x^2 + 2x - 728 = 0$; 26, 28; -28, -26
e) $2W^2 - 98 = 0$; W = 7; L = 14
f) $L^2 - 16L + 48 = 0$; Diag = 13

g) i) $16t^2 - 96t + 144 = 0$ $t = 3s$

ii) $16t^2 - 96t + 140 = 0$ $t = 2.5s$ and $3.5s$

h) $x^2 - 38x + 325 = 0$; $x = 25$, $x - 1 = 24$, $x - 18 = 7$

i) a) $x^2 - 65x + 300 = 0$ $x = 5$

b) $(80 - 2x)(30 - 2x) = 1/2(30)(80)$

$4x^2 - 220x + 1200 = 0$ $x = 6.36$

j) $x^2 - 10x - 144 = 0$ $x = 18$

k) $4(x - 8)(x - 18) = 684$ $x = 27$

l) $x + \frac{1}{x} = 4$; $x^2 - 4x + 1 = 0$ $x = 3.73, 0.27$

m) $L^2 - 8L + 0 = 0$ $L = 8$

n) $P^2 - 6P - 216 = 0$ $P = 18$

OP.VII. PERMUTATIONS AND COMBINATIONS

A-1,2, 3,4

1. Write all the possible permutations of the letters A, T and R. Repetitions of a letter are not permitted.

2. Write all possible three digit numbers that can be formed from the digits 1, 2, 3 and 4. No digit may be repeated within a number.

3. Fasar Analog watches may be obtained as Ladies' or Gents' models; in yellow or white; with a leather or link bracelet; and with 2, 4, 6 or 8 diamonds in the case. Construct a tree diagram to illustrate the possible watches in stock at the factory.

4. A penny, a nickel, a dime and a quarter are tossed consecutively. In how many ways can heads or tails appear?

5. How many 3 digit numbers can be formed using the digits 2, 4, 6, 8 if no digit may be repeated?

6. The football stadium has 4 entrances and 8 exits. In how many ways can Crazy George a) enter and leave the stadium, b) enter and leave by a different exit?

7. In how many ways can four candidates for council president be listed on the ballot?

8. How many more different licence plates can be manufactured if licence plates had to have four letters and two numerals instead of three letters and three numerals?

9. If 9 runners are entered in the 100 m sprint at the Pan Am stadium, how many arrangements of first, second and third place could there be?

10. How many odd numbers consisting of three different digits can be formed from the numerals 1, 2, 3, 4, 5, 6 and 0?

11. In how many ways can a) one die land? b) two dice land?

12. In how many ways can 6 people be seated in a row?

13. In how many ways can 5 men and 5 women be seated in a row, a) if men and ladies must alternate, b) if the ladies must be seated in the first four seats?

14. How many different compositions can be played on a xylophone striking each of eight keys once only?

15. How many different telephone numbers are there on the same exchange if in an exchange the first group of three digits are identical?

A-5,6,7 1. If $5! = 5 \times 4 \times 3 \times 2 \times 1$ and $0! = 1$, evaluate the following:

a) $4!$
b) $8!$
c) $\frac{5!}{(5-2)!}$
d) $\frac{6!}{6!}$
e) $4! \times 3!$
f) $\frac{5! \times 2!}{4!}$
g) $69!$
h) $7! \times 0!$
i) $\frac{3!}{0!}$
j) $6 \times 3!$

2. Why do most calculations with a factorial function not give a read-out for 70!?

3. If $nPr = \frac{n!}{(n-r)!}$ determine

a) ${}_5P_2$
b) ${}_{100}P_2$
c) ${}_{15}P_5$
d) ${}_{10}P_2$
e) ${}_5P_3 \times {}_6P_2$
f) $3! \times {}_5P_1$

4. Using the formula ${}_nP_r = \frac{n!}{(n-r)!}$ calculate the number of 3 different digit numbers that can be formed from the numerals 1, 2, 3, 4, 5, 6 and 7?

5. In how many ways can a president, vice-president, treasurer and secretary be chosen from a group of 6 candidates?

6. Winners for the school raffle prizes of $1000 for first and $500 for second are to be drawn from a barrel containing 8000 ticket stubs. How many possible orders of winners can there be?

7. In how many ways can 10 commercial floats be arranged in the Shannon Festival Parade?

8. In how many ways can the letters of the word "ALTER" be arranged?

9. In how many different ways can AA be arranged if one is coloured red and the other is black?

10. How many arrangements of the letters of the word "ALTAR" are there if the A's are indistinguishable from each other?

11. How many permutations of the letters of each of the following words are there?
    a) EGOTISTICAL
    b) ARKANSAS
    c) SPECIFICITY
    d) LULL

12. How many different arrangements are there of 4 black marbles, 3 white marbles and 5 red marbles?

13. In how many different-looking ways can three sets of identical twins be lined up along the end line of the volleyball court if no one can tell each pair of twins apart and if none of them have identifying numbers or names on their uniforms?

B-1,2

1. Evaluate a) ${}_9C_6$ b) ${}_{15}C_7$ c) ${}_{15}C_8$ d) ${}_{10}C_7$ e) ${}_{10}C_3$

2. In how many ways can 3 people from a group of 8 be chosen to form a committee?

3. How many diagonals does a regular octagon (8 sided) have? (Are some of the segments sides?)

4. The school team, consisting of four curlers, is to be awarded sweaters before the provincial bonspiel. Each sweater is to consist of two of the following colours: blue, orange, white, green, purple and black. How many different combinations of two colours can there be for a sweater?

5. The student council consisting of 8 male and 6 female students wishes to set up a committee consisting of 3 males and 2 females to conduct a survey "Should teenage girls receive preferential driver licence rates?" a) How many such 5 member committees are possible? b) How many such committees would be possible if the council determined that the male members must be in majority?(enrichment)

6. After the basketball game the players dropped in at the nearest fast food outlet, where Wendy, the waitress stated that their Big Hamburgers could have a choice of cheese, lettuce, onions or pickles in a toasted caraway seed bun. How many different orders could there be?

7. How many combinations of 3 letters can be selected from the word "terminal?"

8. How many permutations of 3 letters can be selected from the word "terminal?"

9. Jim Average came to class (again) without pens. Margaret offered him any 2 from her set of 8 pens. In how many ways can he choose two pens to use during class?

10. The volleyball conference consists of 7 teams. Each team plays each other team twice before playoffs. How many volleyball games are played?

11. Dominoe tiles are either blank or marked with 1 to 9 spots in a "double-nine" set. a) How many dominoe tiles are in a double-nine set? b) How many dominoe tiles are in a "double-six" set?

12. How many different 5-card poker hands are possible from an ordinary deck of 52 cards?

13. On a test, students are instructed to answer only 6 questions of 10. What is the greatest number of students who could write the test so that each student has a different set of 6 questions answered?

Answer Key

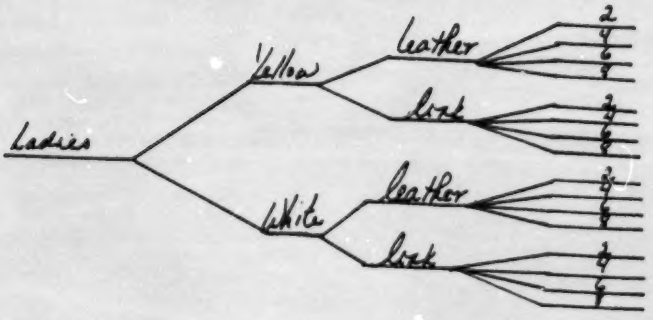
A-1,2, 3,4

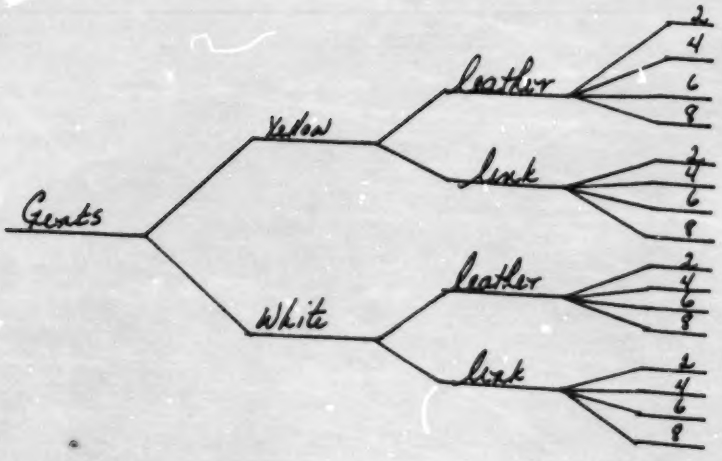
1. ATR, ART, TRA, TAR, RAT, RTA

2.

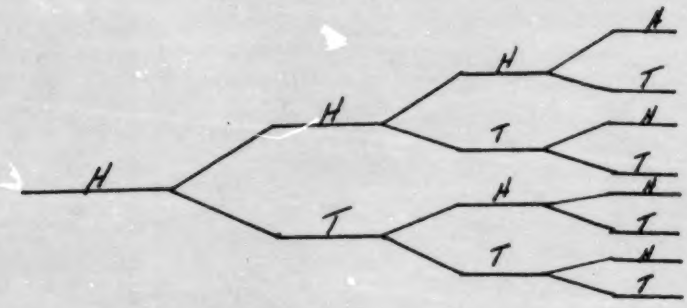
123	234	134	124
132	243	143	142
231	342	341	241
213	324	314	214
312	423	413	412
321	432	431	421

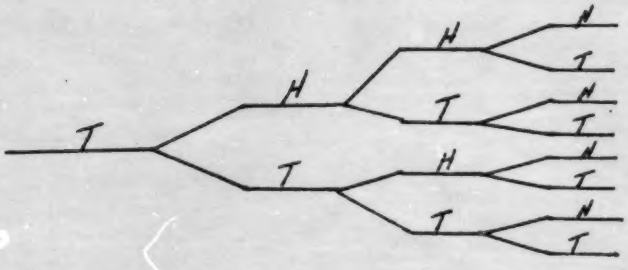
3.





4. Each coin lands either heads or tails.





5. $4 \times 3 \times 2 = 24$ possible three different digit numbers.

6. $4 \times 8 = 32$ ways
   $4 \times 7 = 28$ ways

7. $4 \times 3 \times 2 \times 1 = 24$ ways

8. $26 \times 26 \times 26 \times 26 \times 10 \times 10 = 45\ 697\ 600$
   $26 \times 26 \times 26 \times 10 \times 10 \times 10 = 17\ 576\ 000$
   Difference $= 28\ 121\ 600$

9. $9 \times 8 \times 7 = 504$ arrangements

10. $5 \times 5 \times 3 = 75$
    The final digit must be odd, therefore, 3 choices. The first digit cannot be zero nor the odd digit chosen for the final digit, therefore, five choices. For the middle digit, there is a choice of five since two digits have already been chosen.

11. a) 6
    b) $6 \times 6 = 36$

12. $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

13. a) $\frac{5}{M} \times \frac{4}{L} \times \frac{4}{M} \times \frac{3}{L} \times \frac{3}{M} \times \frac{2}{L} \times \frac{2}{M} \times \frac{1}{L} \times \frac{1}{M} = 2880$

    b) $\frac{4}{L} \times \frac{3}{L} \times \frac{2}{L} \times \frac{1}{L} \times \frac{5}{M} \times \frac{4}{M} \times \frac{3}{M} \times \frac{2}{M} \times \frac{1}{M} = 2880$

14. $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40\ 320$

15. Same for all 10 9 8 7 = 5040

A-5,6,6 1. a) 24
b) 40 320
c) 20
d) 1
e) 144
f) 10
g) $\approx 1.71 \times 10^{98}$
h) 5040
i) 6
j) 36

2. 70! would be a very large number with more than 100 digits in the answer. If the calculator cannot register an exponent greater than 99 the display will print "Error".

3. a) $\frac{5!}{3!} = 20$

   b) $\frac{100!}{98!} = 100 \times 99 = 9900$

   c) $\frac{15!}{10!} = 360\ 360$

   d) $\frac{10!}{6!} = 5040$

   e) $\frac{5!}{2!} \times \frac{6!}{4!} = 1800$

   f) $6 \times \frac{5!}{4!} = 30$

4. ${}_7P_3 = \frac{7!}{(7-3)!} = \frac{7 \times 6 \times 5 \times 4!}{4!} = 210$

5. Since order counts ${}_6P_4 = \frac{6!}{(6-4)!} = 6 \times 5 \times \frac{4 \times 3 \times 2!}{2!} = 360$

6. ${}_{8000}P_2 = \frac{8000!}{(8000-2)!} = \frac{8000!}{7998!} = 8000 \times 7999 \times \frac{7998!}{7998!} = 63\ 984\ 000$

7. Choose 10 out of 10 ${}_{10}P_{10} = \frac{10!}{(10-10)!} = \frac{10!}{0!} = 3\ 628\ 800$

8. ${}_5P_5 = \frac{5!}{0!} = 120$ or $5 \times 4 \times 3 \times 2 \times 1$

9. If the two A's are different, the letters could be arranged in two different ways.

10. $\frac{5!}{2!} = 5 \times 4 \times \frac{3 \times 2 \times 1}{2} = 60$ ways

11. a) $\frac{11!}{2!2!} = 9\ 979\ 200$

    b) $\frac{8!}{3!2!} = 3360$

    c) $\frac{11!}{2!3!} = 3\ 326\ 400$

    d) $\frac{4!}{3!} = 4$

12. $\frac{12!}{4!3!5!} = 27\ 720$

13. $\frac{6!}{2!2!2!} = 90$

B-1,2

1. a) 84 b) 6435 c) 6435 d) 120 e) 120

2. ${}_8C_3 = \frac{8!}{5!3!} = \frac{8 \times 7 \times 6 \times 5}{2 \times 3 \times 5} = 56$ ways or $\frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$

3. Number of segments joining two points ${}_8C_2 = \frac{8!}{6!2!} = 28$
   But 8 of these segments are sides. Therefore, 28-8=20 diagonals.

4. ${}_6C_2 = \frac{6!}{4!2!} = 15$ combinations

5. a) ${}_8C_3 \times {}_6C_2 = \frac{8!}{5!3!} \times \frac{6!}{4!2!} = 840$ committees

   b) ${}_8C_3 \times {}_6C_2 + {}_8C_4 \times {}_6C_1 + {}_8C_5 \times {}_6C_0 = 840 + 420 + 56 = 1316$

6. ${}_4C_0 + {}_4C_1 + {}_4C_2 + {}_4C_3 + {}_4C_4 =$
   Plain ... the works
   $1 + 4 + 6 + 4 + 1 = 16$

7. 56
8. 336
9. 28
10. $2 \cdot {}_7C_2 = 42$
11. a) 55 b) 28
12. 2 598 960
13. 210